

DETERMINING CRACK TIP FIELD PARAMETERS FOR ELASTIC-PLASTIC MATERIALS VIA AN ESTIMATION SCHEME

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This report reviews the theore		estimation scheme which
appeared in the literature. A con		
the elastic-plastic fracture mechanic	-	
geometries. The parameters evalua		
and crack mouth opening displacem		
program listing and an example of	an input and out	cpuc.

FOREWORD

This technical report was prepared by the Aerospace Mechanics Division of the University of Dayton Research Institute. This study was conducted by the authors from November 1979 to November 1980 as a part of the USAF Contract F33615-78-C-5184 with the Air Force Wright Aeronautical Laboratory/Materials Laboratory. The contract, which was initiated under Project No. 2418, Task 24180306, was administered under the direction of the Air Force Materials Laboratory. Dr. Theodore Nicholas of the Metals and Ceramics Division of the Materials Laboratory was the Project Monitor for this study of the application of the Nonlinear Fracture Mechanics (NLFM) parameters to the study of Fatigue Crack Growth.

The authors wish to express their appreciation to Ms. Carol Bruner and Mr. D. Roalef for preparation of the computer program and modules which appear in this report. The methods which appear in this report were used by the authors in computation of analytical NLFM parameters for their fatigue crack growth work.

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SECTION 1 INTRODUCTION

The estimating scheme described in this report is the direct result of prior work by Hutchinson, and co-workers ¹⁻² and by Shih and co-workers ³⁻⁶. This estimating scheme is used to determine a fracture mechanics parameter (the J Integral) which describes the intensity of the Hutchinson⁷, Rice and Rosengren⁸ (HRR) stress-strain field at the crack tip in an elastic-plastic material. The estimating scheme can also be used to determine load-load line displacement and crack mouth opening displacement. The J-Integral parameter determined by the estimating scheme can be used to obtain the crack tip opening displacement when the HRR field singularity describes the crack tip singular behavior.

The two-fold purpose of the report is (a) to review the theoretical basis for the estimation scheme and (b) to describe a computer program which utilizes the estimating scheme to yield values for the J-Integral (J), the load-line displacement (Δ), and the crack mouth opening displacement (CMOD or δ) for five different structural crack geometries.

SECTION 2

THEORETICAL BACKGROUND

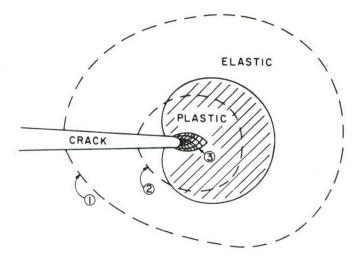
This section of the report has been prepared to provide background information relative to the elastic-plastic fracture mechanics parameters which are derived by the computer program described in Section 3.

2.1 ELASTIC-PLASTIC FIELD PARAMETERS

The solutions for the in-plane tensile opening mode (Mode 1) type of deformation problems are the primary interest in the present work as well as fracture mechanics in general. However, the mathematical difficulties have prevented detailed treatment of elastic-plastic problems. Except in some out-of-plane, tearing mode (Mode 3) type of problems 9,10, rigorous mathematical solutions of elastic-plastic problems are not available in general; the available limited cases of elastic-plastic crack tip stress analyses will be reviewed in this subsection.

Figure 1 shows the crack tip and area ahead of the crack tip 11. The region ahead of the crack-tip is divided into three distinct zones: (1) elastic, (2) elastic-plastic, and (3) intensity non-linear (large strains and rotations, and ductile cavities) zone. The elastic zone (1) controls the behavior when the plastic zone size is small compared to the elastic zone and the geometry. In this case, referred to as small scale yielding, linear elastic fracture mechanics (LEFM) is applicable. If the plastic zone size is large, compared to the case of small scale yielding, LEFM is not applicable.

The intense elastic-plastic stress-strain field contained within zone 2 of Figure 1 is further expanded in Figure 2. When the intensely deformed process zone is small compared to the size of the elastic-plastic zone under consideration, the deformation theory of plasticity for a power hardening material can be used to obtain stress-strain solutions ahead of the crack tip outside the intensely deformed process zone as suggested by Hutchinson 7 and Rice and Rosengren 8 . These authors expressed power hardening using a stress (σ) -strain (ε) relationship given by:



Zone l = An Elastic Field Surrounding the Crack Tip
Zone 2 = An Elastic-Plastic Field Surrounding the Crack Tip

Zone 3 = An Intense Zone of Deformation

Figure 1. Crack-Tip Stress and Strain Fields Surrounding the Crack (Reference 11).

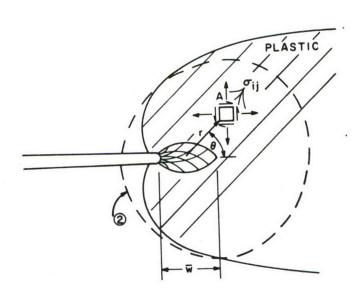


Figure 2. Expanded View of the Elastic-Plastic Stress-Strain Field (Reference 11).

$$\sigma = \sigma_0 \left(\frac{\varepsilon}{\varepsilon_0} \right)^{N} \tag{1}$$

where σ_0 and ϵ_0 are reference stresses and strains, respectively, and N is the strain hardening exponent.

When Equation 1 is used to model the behavior of the material in the plastic range, the stress, strain, and displacement functions for the crack tip region are given by Equation 2, regardless of the amount of plastic deformation:

$$\sigma_{ij} = K_{\sigma} \quad \tilde{\sigma}_{ij} \quad (\theta, N) \cdot r \quad \frac{N}{N+1}$$
 (2a)

$$\varepsilon_{ij} = K_{\varepsilon} \quad \tilde{\varepsilon}_{ij} \quad (\theta, N) \cdot r \quad \frac{1}{N+1}$$
 (2b)

$$u_{i} = K_{\varepsilon} \quad \tilde{u}_{i}(\theta, N) \cdot r^{\frac{1}{N+1}}$$
 (2c)

where K_{σ} and K_{ε} are the magnitudes of singularities of appropriate quantities with K_{ε} being a function of K_{σ} . The functions $\tilde{\sigma}_{ij}$, $\tilde{\epsilon}_{ij}$ and \tilde{u}_{i} depend on angle and exponent N in Equation 1. Equations 2a, 2b and 2c have been referred to as the set of "HRR" field equations after the initial investigators (References 7 and 8).

In the derivation of the HRR field equations, the ${\rm J_2}$ deformation theory of plasticity was used to describe the material behavior. If the loading is proportional, the field solutions obtained using both the ${\rm J_2}$ -deformation theory and the more realistic ${\rm J_2}$ -incremented theory of plasticity are the same. However, when unloading occurs during deformation, the loading path may be different from the assumed proportional loading and the validity of the above solutions is not guaranteed.

The magnitude of the singularity in Equation 2 can be written in terms of J as when the process zone is small (Reference 11):

$$\sigma_{ij} = \sigma_{o} \left(\frac{J}{r\sigma_{o} \epsilon_{o}} \right)^{\frac{N}{N+1}} \tilde{\sigma}_{ij} (\Theta, N)$$
 (3a)

$$\varepsilon_{ij} = \varepsilon_{o} \left(\frac{J}{r\sigma_{o} \varepsilon_{o}} \right)^{\frac{1}{N+1}} \tilde{\varepsilon}_{ij} (\Theta, N)$$
(3b)

$$u_{i} = \varepsilon_{o} \left(\frac{J}{\sigma_{o} \varepsilon_{o}} \right)^{\frac{1}{N+1}} r^{\frac{N}{N+1}} \tilde{u}_{i}(\Theta, N)$$
 (3c)

$$J = \int_{\Gamma} \left[w n_{1} - n_{j} \sigma_{ij} u_{j,i} \right] ds$$

$$W = \int_{0}^{\epsilon_{ij}} \sigma_{ij} d\epsilon_{ij}$$
(4)

In equation 4, Γ is any contour that surrounds the crack tip, ds and n_j are the element length and outward normal log Γ , u_j , i are the displacement gradients and σ_{ij} and ε_{ij} are the stress and strain tensors. The J has been determined to be independent of the location of contour curve Γ .

Hence, for a given cracked material, with the assumption of power law hardening behavior, deformation theory of plasticity, and proportional loading, there exists a unique elastic-plastic stress and strain field which is characterized by its intensity J. In a manner similar to the LEFM approach where the stress intensity factor (K) measures the intensity of stress and strain within the elastic crack tip field, the parameter J defines the intensity of the elastic-plastic crack tip field, the parameter J defines the intensity of the elastic-plastic stress and strain in the crack tip field and thus provides a basis for a nonlinear fracture mechanics approach. The use of J to define the level of elastic-plastic stresses and strains around the crack tip requires that the intensely deformed process zone is small.

For ideally plastic materials, the relationship between the J-integral and the crack tip opening displacement (δ_t) defined by the opening distance between the intercepts of two 45° lines drawn back from the crack tip, with the deformed profile of the stationary crack as shown in Figure 3 is given by References 12 and 13:

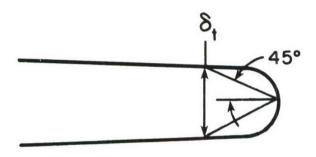


Figure 3. Definition of Crack Tip Opening Displacement $\delta\,.$

$$\delta_{t} = d_{n} \frac{J}{\sigma_{0}} \tag{5}$$

where d_n is a constant. For strain hardening materials, Shih and co-workers (References 4 and 14) have shown that Equation 5 can be used to relate J and δ_t when the constant d_n is replaced with a function which is strongly dependent on the strain hardening exponent and mildly dependent on the ratio σ_0/E (E = Young's modulus). The functional relation for a strain-hardening material is described in Figure 4; Figure 4a applies for plane strain conditions while Figure 4b applies for plane stress conditions. Figure 4 has been derived assuming that Equation 1 describes the material. On the basis of Equations 3 and 5, δ_t , the crack tip opening displacement, is also a parameter which characterizes the intensity of the stress-strain HRR field. The above HRR field equations are applicable only for the case of stationary cracks.

2.2 PARAMETER DETERMINATION

For elastic-plastic materials, the parameters: the J-integral (J), the crack mouth opening displacement (CMOD= δ), and the load-line displacement due to the presence of a crack ($\Delta_{\rm C}$) can be approximated by the contributions due to their linearly elastic and plastic parts (References 3, 4, and 16):

$$J = J^{e} + J^{p}$$

$$\delta = \delta^{e} + \delta^{p}$$

$$\Delta_{c} = \Delta_{c}^{e} + \Delta_{c}^{p}$$
(6)

where superscripts e and p denotes elastic and plastic, respectively.

$$\frac{\varepsilon}{\varepsilon_0} = \alpha \left(\frac{\sigma}{\sigma_0}\right)^n$$
 and $\alpha = 1$

then d_n is as presented in Figure 4. However, if $\alpha \neq 1$ then d_n is equal to the product of the value of d_n given in Figure 4 and the quantity of $\alpha^{1/n}$ (Reference 15).

^{*}If the material is described by the relationship:

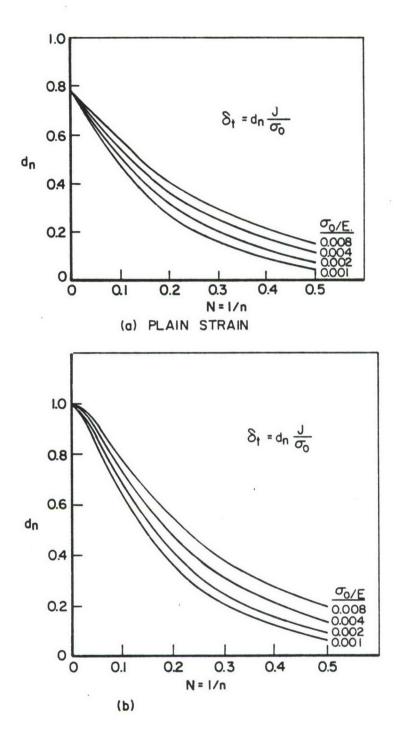


Figure 4. Variation of d with N and σ_0/E (a) Plane Strain (b) Plane Stress (Assuming Equation 1 Applies).

Reasonable estimates of the total load-line displacement (Δ) for the structural geometry can be obtained by summing the contributions due to the presence of the crack, i.e. $\Delta_{\dot{\mathbf{C}}}$ from Equation 6, and to that due to the structural geometry without a crack ($\Delta_{\mathbf{n}}$), (Reference 3):

$$\Delta = \Delta_{\mathbf{C}} + \Delta_{\mathbf{n}} \tag{7}$$

The error in using Equation 7 is small when the distance between the load points is much smaller than other structural dimensions.

The purpose of this subsection is to outline the analysis that must be accomplished in conjunction with Equation 6 to derive estimates of the elastic-plastic parameters. The following three paragraphs present: the elastic formulation, the plastic (strain-hardening) formulation and the elastic-plastic transition formulations, respectively.

2.2.1 Linearly Elastic Contribution

For linearly elastic materials, the elastic crack parameters appearing in Equation 6 can be expressed in the form:

$$\frac{J^{e}}{\sigma_{o} \varepsilon_{o} a} = \left(\frac{\sigma^{\infty}}{\sigma_{o}}\right)^{2} \hat{J}^{e}(a/b)$$

$$\frac{\delta^{e}}{\varepsilon_{o} a} = \left(\frac{\sigma^{\infty}}{\sigma_{o}}\right) \hat{\delta}^{e}(a/b)$$

$$\frac{\Delta^{e}}{\varepsilon_{o} a} = \left(\frac{\sigma^{\infty}}{\sigma_{o}}\right) \hat{\Delta}^{e}_{c}(a/b)$$
(8)

where σ^{∞} is the remotely applied stress, and σ_{0} and ε_{0} are reference stresses and strains related by the expression $\sigma_{0}=E\varepsilon_{0}$. Functions \hat{J}^{e} , $\hat{\delta}^{e}$, and $\hat{\Delta}_{c}^{e}$ are functions only of the ratio of crack length to width (a/b). These functions can be found in Reference 17 for different finite width crack geometries.

2.2.2 Plastic (Strain Hardening) Contribution

To derive the plastic crack parameters given in Equation 6, certain assumptions are required. First, the material is assumed to behave according to a power hardening constitutive $(\sigma-\epsilon)$ equation of the form

$$\frac{\varepsilon}{\varepsilon_0} = \alpha \left(\frac{\sigma}{\sigma_0}\right)^n \tag{9}$$

where α is a dimensionless constant, σ_0 and ε_0 are reference stresses and strains, and n is the hardening exponent. Note that the exponent in Equation 9 is in the inverse of the exponent in Equation 1. For n=1, the material behaves as a linearly elastic material; as n approaches infinity, the material behaves more and more like a perfectly plastic material. Generalizing Equation 9 to multi-axial states via the J₂ deformation theory results in:

$$\frac{\varepsilon_{ij}}{\varepsilon_{o}} = \frac{3}{2} \quad \alpha \left[\frac{\sigma_{e}}{\sigma_{o}}\right]^{n-1} \quad \frac{S_{ij}}{\sigma_{o}}$$
 (10)

where ϵ_{ij} , s_{ij} , and σ_e are the strain deviator, the stress deviator, and the effective stress (= $\sqrt{3/2}$ s_{ij} s_{ij}), respectively. Ilyushin (Reference 8) first noted that the boundary value problems which (1) have an externally applied, monotonically increasing load or displacement and (2) are based on Equation 10, have some special properties. He showed that the stress at each point in the body varies linearly with a single load parameter when tractions are prescribed on all boundaries and the directions of these tractions remain fixed while their magnitudes are everywhere linearly proportional to the load parameter. Since the stress components at each point are proportional, the solution based on J_2 deformation theory also applies for incremental plasticity theory when the load parameter is monotonically increasing.

The functional dependence of the field parameters (stress, strain, and displacement) on the applied load (or displacement) also means that crack tip parameters can be uniquely related to the remotely applied load (σ^{∞}) via the following expressions (References 3 and 4):

$$\frac{J^{p}}{\sigma_{o} \varepsilon_{o} a} = \left(\frac{\sigma_{o}}{\sigma_{o}}\right)^{n+1} \cdot \hat{J}\left(\frac{a}{b}, n\right)$$

$$\frac{\delta^{p}}{\varepsilon_{o} a} = \left(\frac{\sigma_{o}}{\sigma_{o}}\right)^{n} \cdot \hat{\Delta}^{p}\left(\frac{a}{b}, n\right)$$

$$\frac{\Delta^{p}}{\varepsilon_{o} a} = \left(\frac{\sigma_{o}}{\sigma_{o}}\right)^{n} \cdot \hat{\Delta}^{c}\left(\frac{a}{b}, n\right)$$
(11)

where \hat{J}^p , $\hat{\delta}^p$, and $\hat{\Delta}^p_c$ are functions only of $\frac{a}{b}$ and n. The reader will note that the functional forms given by Equation 11 are similar to Equation 8 and, in fact, they reduce to Equation 8 when n=1.

An alternate form of Equation 11 that has been used previously (References 4 and 5) and which is used in the computer program discussed in Section 3 is:

$$J^{P} = \alpha \sigma_{0} \varepsilon_{0} a f_{1}(a/b) \cdot h_{1} \left(\frac{a}{b}, n\right) \cdot \left(\frac{P}{P_{0}}\right)^{n+1}$$

$$\delta^{P} = \alpha \varepsilon_{0} a f_{2}(a/b) h_{2}\left(\frac{a}{b}, n\right) \cdot \left(\frac{P}{P_{0}}\right)^{n}$$

$$\Delta^{P}_{C} = \alpha \varepsilon_{0} a f_{3}(a/b) h_{3}\left(\frac{a}{b}, n\right) \cdot \left(\frac{P}{P_{0}}\right)^{n}$$
(12)

where P is the applied load (per unit thickness), P_0 is the l-imit load (per unit thickness), f_1 , f_2 and f_3 are functions only of geometry and crack length while h_1 , h_2 and h_3 depend on geometry, crack length,

and strain hardening exponent. Shih and co-workers (References 4 and 5) have tabulated the functions h_1 , h_2 , h_3 for a number of geometries for the conditions of plane stress and plane strain. From the reference tabulated data, these functions can be obtained by interpolation for any value within the $\frac{a}{b}$ and n limits given; thus, the plastic (strain-hardening) component of Equation 6 can be computed for any given applied load P (or $\sigma \infty$) from Equation 12 (or 11).

2.2.3 Elastic to Plastic Transition

In the two preceding paragraphs, we reviewed the procedures for estimating the parameters of Equation 6 when the material was either linearly elastic or plastically strain hardening. Under the assumptions for small scale yielding, Bucci, et al. (Reference 16) observed that the transition region between fully linear elastic and fully large-scale plastic deformation could be more accurately modeled if the physical crack length term,i.e. "a" in the elastic components of Equation 6 was replaced with an effective crack length term (a_e) . Bucci, et al. suggested an effective crack length based on adding the Irwin plastic zone size (r_v) to the physical crack length:

$$a_e = a + r_y \tag{13}$$

where

$$r_{y} = \frac{1}{\beta \pi} \left(\frac{K}{\sigma_{o}} \right)^{2} \tag{14}$$

with β = 2 for plane stress and β = 6 for plane strain. The stress intensity factor is represented by K.

Subsequently, Shih, et al. (Reference 4) defined the effective crack length with a modification to account for strain hardening and for the relationship of the load to limit load. The Shih, et al. formulation for the effective crack length is:

$$a_e = a + \phi r_y \tag{15}$$

where

$$r_{y} = \frac{1}{\beta\pi} \left(\frac{n-1}{n+1} \right) \left(\frac{K}{\sigma_{0}} \right)^{2}$$
 (16)

and

$$\phi = \frac{1}{1 + (P/P_{O})^{2}} \tag{17}$$

The factor ϕ provides a correction to r_y in the case of small scale yielding, and also limits the contribution from the plastically adjusted crack length for large-scale yielding so that the values of $\frac{J}{J^p}$, $\frac{\delta}{\delta^p}$, and $\frac{\Delta_C}{\Delta_C^p}$ approach unity.

The version of Equation 6 embedded in the computer program discussed in Section 3 is given by:

$$J = J^{e}(a_{e}) + J^{p}(a,n)$$

$$\delta = \delta^{e}(a_{e}) + \delta^{p}(a,n)$$

$$\Delta_{c} = \Delta_{c}^{e}(a_{e}) + \Delta_{c}^{p}(a,n)$$
(18)

where a_e is the effective crack length defined in Equation 15. The elastic contribution given by Equation 8 is thus modified by the replacement of the physical crack length by the effective crack length. The plastic contribution is given by Equation 12.

2.3 FORMULATIONS FOR DIFFERENT GEOMETRIES

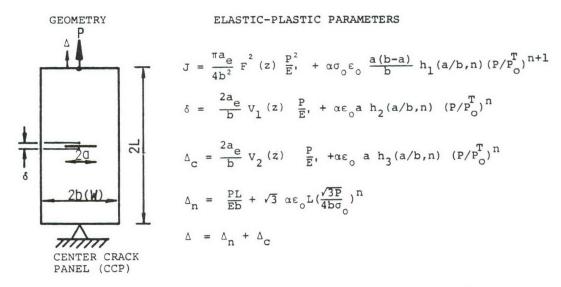
This subsection has been designed to present all the relevant information and equations that would allow one to use the estimating procedure outlined in subsection 2.2 for five different structural geometries. The equations and data presented herein have been previously presented by others (References 2 through 5 and 17) and are only repeated here for completeness. The information for each structural geometry is summarized using a figure and two supporting tables. The five structural geometries considered are listed in Table 1 along with the individual figure and table numbers which present the supporting information.

Each of the five supporting figures all are arranged in the same way so that each geometry's relevant equations and data appear in the same location in the figures. The arrangement of the figures is such that a schematic of the geometry appears in the upper left hand side and the individual versions of Equation 18 (the elastic-plastic parameters) can be found in the upper right hand side. The remainder of the figure is devoted to presenting equations that further define the elastic and plastic components of Equation 18 and to defining the tables that contain discrete value information on the plastic functions h_1 , h_2 , and h_3 .

In order to ensure that the information in Figures 5 through 9 is properly interpolated, a few additional notes are presented here. First, E' is Young's modulus (E) for plane stress and is equal to $E/(1-\nu^2)$ for plane strain where ν is the Poisson's ratio of the material. The loads, P and P_0^T , are the applied load and the theoretical limit load for a perfectly plastic material ($n=\infty$) respectively; both are expressed per unit thickness.

TABLE 1

LIST OF THE CRACK GEOMETRIES CONSIDERED AND THE FIGURES AND TABLES WHICH SUPPORT THEIR ANALYSIS	SIDERED A THEIR AN	ND THE F ALYSIS	IGURI	ES AN	<u>e</u>
STRUCTURAL CRACK GEOMETRY	SUPPORT	SUPPORTING FIGURES AND TABLES	RES 1	AND 7	ABLES
CENTER CRACK PANEL	Figure	Figure 5, Tables 2a and 2b	s 2a	and	2b
COMPACT TENSION CRACK	Figure	Figure 6, Tables 3a and 3b	s 3a	and	3b
DOUBLE EDGE CRACKED PANEL	Figure	Figure 7, Tables 4a and 4b	s 4a	and	4b
SINGLE EDGE CRACKED PANEL LOADED IN THREE-POINT BENDING	Figure	Figure 8, Tables 5a and 5b	s 5a	and	55
SINGLE EDGE CRACKED PANEL LOADED IN TENSION	Figure	Figure 9, Tables 6a and 6b	s 6a	and	q 9



SUPPORTING ELASTIC FUNCTIONS: F, V_1 AND V_2 with $z = a_e/b$

$$F(z) = \sqrt{\sec \left(\frac{\pi z}{2}\right)}$$

$$V_1(z) = -0.071 - 0.535z + 0.169z^2 + 0.02z^3 - 1.071(1/z) \ln(1-z)$$

$$V_2(z) = -1.071 + 0.250z - 0.357z^2 + 0.121z^3 - 0.047z + 0.008z^5$$

$$-1.071(1/z) \ln(1-z)$$

THEORETICAL PLASTIC LIMIT LOAD: P_{O}^{T}

Plane Stress Condition

$$P_0^T = \frac{4}{3} \sigma_0 (b-a)$$

Plane Strain Condition

$$p_0^T = 2\sigma_0 \quad (b-a)$$

SUPPORTING PLASTIC FUNCTIONS: h₁, h₂, and h₃

Plane stress condition tabularized in Table 2a

Plane strain condition tabularized in Table 2b

Figure 5. Elastic-Plastic Parameters for the Center-Cracked Panel (References 4, 5 and 17).

TABLE 2a

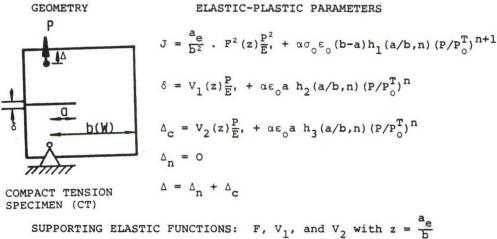
 h_1 , h_2 and h_3 FOR THE PLANE STRESS CCP IN TENSION (REFERENCE 5).

	n = 1	n = 2	n = 3	n = 5	n = 7	n = 10	n = 13	n = 16	n = 20
ٿ	2.544	2.972	3.140	3.195	3.106	2.896	2.647	2.467	2.196
$a/b = 1/4 \ h_2$	3.116	3.286	3.304	3.151	2.926	2.595	2.288	2.081	1.814
h ₃	0.611	1.010	1.352	1.830	2.083	2.191	2.122	2.009	1.792
, ,									
L _P	2.344	2.533	2.515	2.346	2.173	1.953	1.766	1.608	1.431
$a/b = 3/8 \ h_2$	2.710	2.621	2.414	2.032	1.753	1.473	1.279	1.134	0.988
لم ا	0.807	1.195	1.427	1.594	1.570	1.425	1.267	1.133	0.994
`							-		
h	2.206	2.195	2.057	1.809	1.632	1.433	1.300	1.174	1.000
$a/b = 1/2 \ h_2$	2.342	2.014	1.703	1.299	1.071	0.871	0.757	999.0	0.557
h ₃	0.927	1.186	1.256	1.178	1.040	0.867	0.758	0.668	0.560
•									
h	2.115	1.912	1.690	1.407	1.221	1.012	0.853	0.712	0.573
$a/b = 5/8 \ h_2$	1.968	1.458	1.126	0.785	0.617	0.474	0.383	0.313	0.256
h ₃	0.975	1.053	0.970	0.763	0.620	0.478	0.386	0.318	0.273
, ,									
h	2.073	1.708	1.458	1.208	1.082	0.956	0.745	0.646	0.532
$a/b = 3/4 \ h_2$	1.611	0.970	0.685	0.452	0.361	0.292	0.216	0.183	0.148
(h ₃	0.933	0.802	0.642	0.450	0.361	0.292	0.216	0.183	0.149

TABLE 2b

 h_1 , h_2 AND h_3 FOR THE PLANE STRAIN CCP IN TENSION (REFERENCE 5).

	n = 1	n = 2	n = 3	n = 5	n = 7	n = 10	n = 13	n = 16	n = 20
لًا	2.535	3.009	3.212	3.289	3.181	2.915	2.625	2.340	2.028
$a/b = 1/4 \left\langle h_2 \right\rangle$	2.680	2.989	3.014	2.847	2.610	2.618	1.971	1.712	1.450
h ₃	0.536	0.911	1.217	1.639	1.844	1.554	1.802	1.637	1.426
, ,									
L ₁	2.344	2.616	2.648	2.507	2.281	1.969	1.709	1.457	1.193
$a/b = 3/8 \left\{ h_2 \right\}$	2.347	2.391	2.230	1.876	1.580	1.276	1.065	0.890	0.715
h ₃	0.699	1.059	1.275	1.440	1.396	1.227	1.050	0.888	0.719
h	2.206	2.291	2.204	1.968	1.759	1.522	1.323	1.155	0.978
$a/b = 1/2 h_2$	2.028	1.856	1.600	1.230	1.002	0.799	0.664	0.564	0.466
۳ - ۱	0.803	1.067	1.155	1.101	0.968	0.796	0.665	0.565	0.469
,									
u u	2.115	1.960	1.763	1.616	1.169	0.863	0.628	0.458	0.300
$a/b = 5/8 \ h_2$	1.705	1.322	1.035	969.0	0.524	0.358	0.250	0.178	0.114
h 3	0.844	0.937	0.879	0.691	0.522	0.361	0.251	0.178	0.115
, ,									
h h	2.072	1.732	1.471	1.108	0.895	0.642	0.461	0.337	0.216
$a/b = 3/4 \ h_2$	1.345	0.857	0.596	0.361	0.254	0.167	0.114	0.081	0.051
h ³	0.805	0.700	0.555	0.359	0.254	0.168	0.114	0.081	0.052



SUPPORTING ELASTIC FUNCTIONS: F,
$$V_1$$
, and V_2 with $z = \frac{e}{b}$

$$F(z) = \frac{(2+z)}{\sqrt{z}(1-z)^{1.5}} (0.886+4.64z-13.32z^2+14.72z^3-5.6z^4)$$

$$V_1(z) = (5.435+43.315z-83.166z^2+57.694z^3)/(1-z^2)$$
and

 $V_2(z) = (0.995+27.977z-27.209z^2+11.062z^3)/(1-z^2)$

THEORETICAL PLASTIC LIMIT LOAD: POT

Plane Stress Condition

$$P_0^T = 1.072\eta \ (b-a)\sigma_0$$

Plane Strain Condition

$$P_0^T = \frac{2.52}{\sqrt{3}} \eta \ (b-a) \sigma_0$$

with
$$\eta = \sqrt{\left(\frac{2a}{b-a}\right)^2 + 2\left(\frac{2a}{b-a}\right) + 2} - \left(\frac{2a}{b-a} + 1\right)$$

SUPPORTING PLASTIC FUNCTIONS: h1, h2, h3

Plane stress condition summarized in Table 3a Plane strain condition summarized in Table 3b

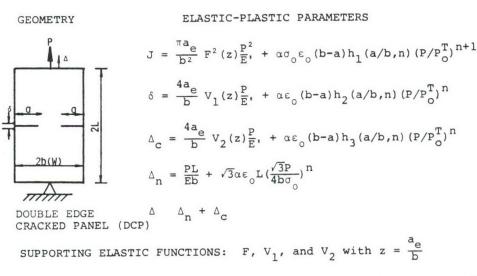
Figure 6. Elastic-Plastic Parameters for the Compact Tension Specimen (References 4, 5 and 17).

 h_1 , h_2 AND h_3 FOR THE PLANE STRAIN CCP IN TENSION (REFERENCE 5). TABLE 2b

	n = 1	n = 2	n = 3	n = 5	n = 7	n = 10	n = 13	n = 16	n = 20
4	1.609	1.464	1.284	1.060	0.903	0.729	0.601	0.511	0.395
$a/b = 1/4 \left\{ h_2 \right\}$	17.552	12.042	10.706	8.736	7.316	5.744	4.629	3.746	2.916
h ₃	9.670	7.996	7.205	5.944	5.000	3.945	3.191	2.591	2.023
) /									
Lh.	1.552	1.249	1.047	0.801	0.647	0.484	0.377	0.284	0.220
$a/b = 3/8 \left\{ h_2 \right\}$	12.410	8.203	6.538	4.563	3.447	2.442	1.830	1.360	1.019
h ₃	7.800	5.734	4.615	3.253	2.475	1.765	1.330	0.990	0.746
,									14
ر _ا	1.398	1.084	0.901	0.686	0.558	0.436	0.356	0.298	0.238
$a/b = 1/2 \left\{ h_2 \right\}$	9.155	5.673	4.212	2.801	2.123	1.571	1.245	1.026	0.814
h ₃ -	6.288	4.149	3.107	2.087	1.590	1.181	0.938	0.774	0.614
s ,									
ط	1.274	1.031	0.875	0.695	0.593	0.494	0.423	0.370	0.310
$a/b = 5/8 \left\{ h_2 \right\}$	7.471	4.483	3.347	2.367	1.923	1.539	1.292	1.116	0.928
h ₃	5.419	3.375	2.536	1.804	1.468	1.176	0.988	0.853	0.710
, h	1.234	0.977	0.833	0.683	0.598	0.506	0.431	0.373	0.314
$a/b = 3/4 \left\{ h_2 \right\}$	6.252	3.780	2.893	2.135	1.775	1.437	1.204	1.030	0.857
h ₃	4.767	2.922	2.242	1.657	1.379	1.116	0.936	0.800	999.0
,									
h,	1.133	1.010	0.775	0.680	0.650	0.620	0.490	0.470	0.420
$a/b + 1 \left\{ h_2 \right\}$	5.288	3.536	2.412	1.905	1.734	1.592	1.232	1.166	1.029
h 3	4.231	2.829	1.930	1.524	1.387	1.274	0.985	0.933	0.824

TABLE 3b

14.563 0.248 0.345 0.665 20 1.566 0.370 0.485 0.514 10.887 0.266 0.317 0.236 0.368 0.630 1.136 0.098 0.909 11 5). 1.325 0.448 12.570 0.176 0.216 0.393 0.568 0.686 0.530 0.347 0.686 0.730 1,333 1.066 STRAIN COMPACT TENSION SPECIMEN (REFERENCE 9.371 0.494 0.887 = u = 13 1.258 11.460 8.517 0.276 1.102 0.793 0.314 0.788 0.585 0.459 0.983 0.746 0.575 1.124 0.869 0.850 1.573 1.258 = 10 10.745 1.248 7.942 0.443 0.602 1.319 0.950 1.794 1.292 1.199 1.000 1.795 0.888 0.717 1.114 1.436 0.461 1.441 10.538 1.876 1.080 0.693 2.970 0.685 1.912 1.413 1.810 1.334 7.706 0.752 1.373 0.864 1.450 1.697 11 4.319 0.919 1.475 10.788 0.970 1.180 0.974 2.359 2.433 1.946 2 7.774 3.103 2.747 2.024 1.787 1.033 2.337 1.807 11 11.675 8.170 4.643 1.242 3.423 2.583 3.179 1.350 1.783 1.392 6.521 4.304 1.237 1.263 2.456 3.093 2.474 3 3.157 II h, h2 AND h3 FOR THE PLANE 2.048 1.716 8.176 1.509 5.846 1.449 5.760 4.572 1.450 8.506 3.948 3.738 2.990 4.268 1.424 3.048 2 12.481 3.431 11 2.227 2.148 12.644 7.944 1.709 1.935 1.763 7.612 1.568 4.310 9.852 6.406 6.370 5.388 -9.327 5.521 4.857 h2 h3 h₁ h₂ h₁
h₂
h₃ h₂ h₃ h₂ h₁ a/b = 1/4a/b = 3/8a/b = 1/2a/b = 5/8a/b = 3/4a/b + 1



$$F(z) = (1.122 - 0.561z - 0.205z^{2} + 0.471z^{3} - 0.190^{4})/\sqrt{1-z}$$

$$V_{1}(z) = (\frac{2}{\pi z}) [0.459 (\sin \frac{\pi z}{2}) - 0.065 (\sin \frac{\pi z}{2})^{3} - 0.007 (\sin \frac{\pi z}{2})^{5} + \cosh^{-1} (\sec \frac{\pi z}{2})]$$

$$V_{2}(z) = (\frac{2}{\pi z}) [0.0629 - 0.0610 (\cos \frac{\pi z}{2})^{4} - 0.0019 (\cos \frac{\pi z}{2})^{8} + \ln (\sec \frac{\pi z}{2})]$$

THEORETICAL PLASTIC LIMIT LOAD: P_0^T

Plane Stress Condition

$$P_0^T = \frac{4}{\sqrt{3}}\sigma_0 \quad (b-a)$$

Plane Strain Condition

$$P_0^T = 5.94 \sigma_0 \text{ (b-a)}$$

SUPPORTING PLASTIC FUNCTIONS: h_1 , h_2 , and h_3

Plane stress condition tabulated in Table 4a Plane strain condition tabulated in Table 4b

Figure 7. Elastic-Plastic Parameters for Double-Edge Cracked Panel (References 4, 5 and 17).

TABLE 4a

 h_1 , h_2 AND h_3 FOR THE PLANE STRESS DECP IN TENSION (REFERENCE 5).

		n = 1	n = 2	n = 3	n = 5	7 = n .	n = 10	n = 13	n = 16	n = 20
	h	1.01100	1.22620	1.35600	1.48280	1.54340	1.57770	1.59370	1.59080	1.58790
a/b = 1/4	h >	1.72580	1.81860	1.88590	1.91670	1.90480	1.85350	1.80190	1.74630	1.69980
	h 3	0.29565	0.53672	0.76993	1.16890	1.48990	1.81520	2.02200	2.12440	2.19810
	, ,								Sec	
	4	1.29310	1.41760	1.42740	1.34110	1.23740	1.09370	0.96971	0.87309	0.67438
a/b = 3/8	4 h2	2.59390	2.39340	2.22100	1.86440	1.58780	1.28340	1.06810	0.92165	0.70934
	h ₃	0.65822	1.03710	1.29460	1.52010	1.54690	1.41240	1.22720	1.06810	0.82951
	(1								١
	h_	1.47460	1.46570	1.37830	1.16790	1.01040	0.84503	0.73169	0.62526	0.20790
a/b = 1/2	4 h ₂	3.51420	2.82070	2.33660	1.66980	1.27680	0.94377	0.76195	0.63010	0.23176
	h ₃	1.18380	1.58120	1.69130	1.56290	1.31980	0.08000	0.80855	0.66195	0.26553
	(
	h	1.58600	1.45380	1.28450	1.03760	0.88209	0.73733	0.64944	0.46645	0.02024
a/b = 5/8	4 h2	4.55860	3.14460	2.31800	1.44890	1.06070	0.79048	0.65694	0.47331	0.02766
	^ه	1.93220	2.13830	1.94990	1.44400	1.09380	0.80888	0.66455	0.48664	0.03169
	(
	4	1.65200	1.42590	1.11760	0.97914	0.83350	0.70092	0.63039	0.29662	90000.0
a/b = 3/4	4 h2	5.89560	3.37110	2.21450	1.29740	0.96595	0.74142	0.63559	0.31176	0.00013
	h ₃	3.06300	2.67070	2.06130	1.31380	0.97799	0.74708	0.63801	0.31834	0.00016

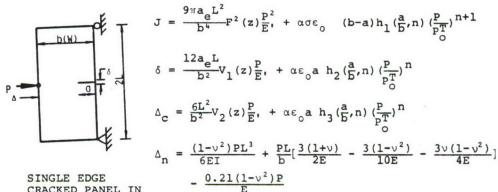
TABLE 4b

 ${
m h_1}$, ${
m h_2}$ AND ${
m h_3}$ FOR THE PLANE STRAIN DECP IN TENSION (REFERENCE 5).

		n = 1	n = 2	n = 3	n = 5	n = 7	n = 10	n = 13	n = 16	n = 20
	h,	5.01	128.40	28.67	155.43	749.07	7530.80	75618.00	741530.00	15600000.00
a/b = 1/4	h >	3.33	7.41	16.55	78.46	362.04	3507.80	34044.00	326360.00	6719300.00
	h 2	0.57	2.20	6.82	49.33	297.75	3720.90	43022.00	459780.00	10421000.00
	, ,									
	4	6.41	14.56	30.42	120.99	461.24	3354.20	20374.00	168810.00	2278000.00
a/b = 3/8	h >	2.00	19.6	18.64	67.83	244.20	1676.30	11491.00	80300.00	1062000.00
	(h3	1.27	4.22	11.17	59.32	266.53	2211.50	16698.00	123400.00	1693100.00
	h,	7.31	14.59	27.07	87.53	275.57	1505.90	8109.40	43102.00	343780.00
a/b = 1/2	h >	6.78	11.22	18.94	53.76	155.16	784.89	4008.90	20719.00	173640.00
	h ₃	2.28	6.39	14.29	55.49	187.17	1051.10	5610.10	29420.00	244640.00
	,									
	ت	7.87	13.40	21.62	53.51	128.26	455.26	1558.40	5376.80	28037.00
a/b = 5/8	h >	8.79	11.91	16.73	34.22	73.40	238.50	783.60	2623.40	13141.00
	h ₃	3.73	8.28	14.83	38.63	91.15	310.61	1030.70	3458.70	17316.00
	,									
	h L	8.19	12.40	17.72	33.06	55.12	105.32	221.79	518.00	894.59
a/b = 3/4	h >	11.37	12.21	14.07	20.72	31.92	58.76	117.99	262.33	473.82
	h ₃	5.91	9.89	13.89	23.82	38.01	70.98	142.31	315.28	575.54

GEOMETRY

ELASTIC-PLASTIC PARAMETERS



CRACKED PANEL 15

THREE-POINT BENDING $\Delta = \Delta_n + \Delta_c$

$$= \Delta_n + \Delta_c$$

SUPPORTING ELASTIC FUNCTIONS: F, V_1 , and V_2 with $z = \frac{a_e}{b}$ $F(z) = 1.09 - 1.735z + 8.2z^2 - 14.18z^3 + 14.57z^4$ $V_1(z) = 0.76 - 2.28z + 3.87z^2 - 2.04z^3 + 0.66/(1-z)^2$ $V_2(z) = (\frac{z}{1-z})^2 (5.58 - 19.57z + 36.82z^2 - 34.94z^3 + 12.77z^4)$ where z = a/b

THEORETICAL PLASTIC LIMIT LOAD: PT

PLANE STRESS CONDITION

$$P_0^T = 0.536\sigma_0 (b-a)^2/L$$

PLANE STRAIN CONDITION

$$P_0^T = 0.728\sigma_0 (b-a)^2/L$$

SUPPORTING PLASTIC FUNCTIONS: h1, h2, h3

Plane stress condition summarized in Table 5a Plane strain condition summarized in Table 5b

Figure 8. Elastic-Plastic Parameters for the Single-Edge Cracked Panel in Three-Point Bending (References 4, 5 and 17).

TABLE 5a

 h_1 , h_2 AND h_3 FOR THE PLANE STRESS SECP UNDER THREE-POINT BENDING (REFERENCE 5).

	n = 1	n = 2	n = 3	n = 5	n = 7	n = 10	n = 13	n = 16	n = 20
4	0.869	0.731	0.629	0.479	0.370	0.246	0.174	0.117	0.059
$a/b = 1/4 \ h_2$	5.690	4.503	3.680	2.614	1.947	1.290	0.897	0.603	0.307
(h ₃	4.007	8.812	7.189	4.731	3,388	2.204	1.517	1.012	0.508
,									
4	0.963	0.797	0.680	0.527	0.418	0.307	0.232	0.174	0.105
$a/b = 3/8 \left\{ h_{y} \right\}$	5.085	3.732	2.929	2.071	1.580	1.134	0.841	0.626	0.381
(h3	4.420	5.533	4.482	3.172	2.409	1.726	1.277	0.948	0.575
,									
ڇ		0.767	0.621	0.453	0.324	0.202	0.128	0.081	0.030
$a/b = 1/2 \left\{ h_{e} \right\}$	-	3.120	2.320	1.547	1.077	0.655	0.410	0.259	0.097
(h ₃		4.085	3.092	2.081	1.442	0.874	0.545	0.344	0.129
,									
4	-	0.786	0.649	0.494	0.357	0.235	0.173	0.105	0.047
$a/b = 5/8 \left\{ h, \right\}$	4.551	2.830	2.118	1.455	1.023	0.656	0.472	0.286	0.130
h ₃	4.617	3.434	2.599	1.794	1.258	0.803	0.577	0.349	0.158
ط	1.067	0.786	0.643	0.474	0.343	0.230	0.167	0.110	0.044
$a/b = 3/4 \left\{ h \right\}$		2.656	1.967	1.329	0.928	0.601	0.427	0.280	0.114
	4.394	3.012	2.235	1.510	1.052	0.680	0.483	0.316	0.129
,									

TABLE 5b

 h_1 , h_2 AND h_3 FOR THE PLANE STRAIN SECP UNDER THREE-POINT BENDING (REFERENCE 5).

	n = 1	n = 2	n = 3	n = 5	n = 7	n = 10	n = 13	n = 16	n = 20
ٿ	1.203	1.034	0.930	0.762	0.633	0.523	0.396	0.303	0.215
$a/b = 1/4 \left\{ h \right\}$	5 5.799	4.665	4.006	3.080	2.454	1.934	1.446	1.088	0.758
رياً .	3 4.083	9.726	8.362	5.863	4.466	3.421	2.542	1.901	1.318
٠			0.0	0					0
_ 			1.018	0.840	0.695	0.556	0.442	0.360	0.265
a/b = 3/8 h		3.931	3.199	2.384	1.927	1.471	1.153	0.928	0.684
(h ₃			5.031	3.737	3.016	2.302	1.803	1.452	1.070
,									
ٹ	1.409	1.094	0.922	0.675	0.495	0.331	0.211	0.135	0.074
$a/b = 1/2 \left\{ h \right\}$		3.283	2.527	1.868	1.192	0.773	0.480	0.304	0.165
(h ₃	3 4.687	4.332	3.489	2.352	1.662	1.079	699.0	0.424	0.230
, (
٣	1.456	1.070	968.0	0.631	0.436	0.255	0.142	0.084	0.041
a/b = 5/8 h		2.861	2.156	1.369	0.907	0.518	0.287	0.166	0.081
(h ₃	3 4.705	3.490	2.700	1.722	1.142	0.652	0.361	0.209	0.102
ے	1.477	1.145	0.974	0.693	0.500	0.348	0.223	0.140	0.075
a/b = 3/4 h		2.754	2.096	1.361	0.936	0.618	0.388	0.239	0.127
(h ₃	3 4.491	3.141	2.404	1.556	1.068	0.704	0.441	0.272	0.144
,		-		-					

GEOMETRY
$$J = \frac{\pi a_e F^2(z) P^2}{E'b^2} + \alpha \sigma_o \varepsilon_o \frac{a(b-a)}{b} h_1 \quad (a/b,n) \left(\frac{P}{pT}\right)^{n+1}$$

$$\delta = \frac{4a_e V_2(z) P}{bE'} + \alpha \varepsilon_o a h_3 \quad (a/b,n) \left(P/P_o^T\right)^n$$

$$\Delta_c = \frac{4a_e V_1(z) P}{bE'} + \alpha \varepsilon_o a h_3 \left(a/b,n\right) \left(P/P_o^T\right)^n$$

$$\Delta_n = \frac{2PL}{Eb} + \sqrt{3}\alpha \varepsilon_o L \left(\frac{\sqrt{3}P}{2b\sigma_o}\right)^n$$
SINGLE EDGE CRACKED PANEL IN TENSION (SET) $\Delta = \Delta_n + \Delta_c$

SUPPORTING ELASTIC FUNCTIONS: F, V_1 and V_2 with $z = \frac{a_e}{b}$

$$F(z) = \sqrt{(2/\pi z) Tan(\pi z/2)} [0.752 + 2.02z + 0.37\{1 - Sin(\pi z/2)\}]$$

F(z) = $\sqrt{(2/\pi z) \text{Tan}(\pi z/2)} [0.752 + 2.02z + 0.37\{1 - \sin(\pi z/2)\}^3]$ Sec($\pi z/2$) $V_1(z) = [1.46 + 3.42\{1 - \cos(\pi z/2)\}] \text{ (Sec } \pi z/2)^2$ $V_2(z) = [z/(1-z)^2][0.99 - z(1-z)(1.3 - 1.2z + 0.7z^2)]$

THEORETICAL PLASTIC LIMIT LOAD: P_{\bigcirc}^{T}

PLANE STRESS CONDITION

$$P_0^T = 1.072 \text{ n (b-a)} \sigma_0$$

PLANE STRAIN CONDITION

$$P_0^T = \frac{2.52}{\sqrt{3}} \eta \ (b-a) \sigma_0$$

with
$$\eta = \sqrt{\left(\frac{a}{b-a}\right)^2 + 1} - \left(\frac{a}{b-a}\right)$$

SUPPORTING PLASTIC FUNCTIONS: h1, h2, h3

Plane stress condition summarized in Table 6a Plane strain condition summarized in Table 6b

Figure 9. Elastic-Plastic Parameters for Single-Edge Cracked Panel in Tension (References 4, 5 and 17).

TABLE 6a $$h_{1},\ h_{2},\ AND\ h_{3}$ FOR THE PLANE STRESS SECP IN TENSION (REFERENCE 5).

	n = 1	n = 2	n = 3	n = 5	n = 7	n = 10	n = 13	n = 16	n = 20
س	3.140	3.261	2.919	2.115	1.531	0.960	0.615	0.400	0.230
$a/b = 1/4 \ h_2$	4.672	4.300	3.695	2.532	1.755	1.053	0.656	0.419	0.237
. h ₃	10.090	6.488	4.362	2.185	1.239	0.630	0.362	0.224	0.123
, ,									
г Г	2.809	2.365	1.943	1.367	1.009	0.677	0.474	0.342	0.226
$a/b = 3/8 \ h_2$	4.465	3.426	2.632	1.685	1.181	0.762	0.524	0.372	0.244
h ₃	5.047	2.653	1.604	0.812	0.525	0.328	0.223	0.157	0.102
, ,									
-μ ⁻	2.459	1.665	1.254	0.776	0.510	0.286	0.164	960.0	0.047
$a/b = 1/2 \ h_2$	4.369	2.726	1.909	1.093	0.694	0.380	0.216	0.124	0.061
h ₃	3.095	1.429	0.871	0.461	0.286	0.155	0.088	0.051	0.025
, ,									
h	2.070	1.408	1.105	0.755	0.551	0.363	0.248	0.172	0.107
$a/b = 3/8 \left\{ h_2 \right\}$	4.297	2.552	1.837	1.160	0.816	0.523	0.353	0.242	0.150
h ₃	2.270	1.127	0.771	0.478	0.336	0.215	0.146	0.100	0.062
, ,									
Lh]	1.696	1.142	0.910	0.624	0.447	0.280	0.181	0.118	0.067
$a/b = 3/4 \ h_2$	4.240	2.468	1.805	1.147	0.798	0.490	0.314	0.203	0.115
(h3	1.983	1.087	0.784	0.494	0.344	0.211	0.136	0.058	0.050

Table 6b $$h_{1},\ h_{2},\ \mbox{AND }h_{3}$ for the plane strain secp in Tension (Reference 5).

	n = 1	n = 2	n = 3	n = 5	n = 7	n = 10	n = 13	91 = u	n = 20
<u>ب</u>		4.767	4.639	3.815	3.056	2.170	1.548	1.105	0.712
$a/b = 1/4 \ h_2$	-	4.559	4.281	3.391	2.639	1.808	1.253	0.875	0.552
_ h ₃	10.271	7.635	5.873	3.695	2.483	1.496	0.970	0.654	0.405
, ,									
L _P	3.81	3.250	2.626	1.680	1.064	0.539	0.276	0.142	0.060
$a/b = 3/8 \ h_2$		3.493	5.669	1.571	0.946	0.458	0.229	0.116	0.048
(h ₃		2.992	1.904	0.923	0.515	0.240	0.119	090.0	0.025
, (
h	3.398	2.302	1.694	0.928	0.514	0.213	0.090	0.039	0.012
$a/b = 1/2 \left\{ h_2 \right\}$		2.765	1.888	0.954	0.507	0.204	0.085	0.036	0.011
h ₃		1.537	0.912	0.417	0.215	0.085	0.036	0.015	0.004
, ,									
h		1.795	1.299	0.697	0.378	0.153	0.063	0.026	0.008
$a/b = 5/8 \ h_2$		2.439	1.622	908.0	0.423	0.167	0.067	0.027	0.008
h ₃	2.311	1.084	0.681	0.329	0.171	0.067	0.027	0.011	0.003
, (
ر _ا		1.607	1.245	0.769	0.477	0.233	0.116	0.059	0.021
$a/b = 3/4 \ h_2$		2.515	1.789	1.027	0.619	0.296	0.146	0.073	0.027
(h ₃		1.104	0.765	0.435	0.262	0.125	0.062	0.031	0.011

SECTION 3 COMPUTER PROGRAM DESCRIPTION

The purpose of this section is to describe the computer program (EST) listed in Appendix A. The EST computer program was written in FORTRAN IV for interactive use on the CYBER 175. The first subsection describes its function and the major elements of the program using flow diagrams. The second subsection describes the program output and various methods for presenting the output data. The third subsection presents the input and how one should prepare the material properties data. The final subsection describes the format of Appendix A.

3.1 PROGRAM FUNCTION

The purpose of the EST computer program is to calculate the value of the J-Integral (J), the crack mouth opening displacement (CMOD = δ), and the load-line displacement (Δ), as well as their elastic-plastic components. The material is assumed to behave according to Ramberg-Osgood type of stress-strain relationship:

$$\frac{\varepsilon}{\varepsilon_0} = \frac{\sigma}{\sigma_0} + \alpha \left(\frac{\sigma}{\sigma_0}\right)^n \tag{19}$$

where ε_0 and σ_0 are reference strains and stresses related by $\varepsilon_0 = \sigma_0/E$, E is the elastic modulus, and α and n are power hardening constants. The program calculates the parameters according to the estimating procedures defined within subsection 2.2 and uses the equations and data presented in Figures 5 through 9 and Tables 2 through 6 in subsection 2.3. The values of functions h_1 , h_2 and h_3 are known for the particular exponent n and crack length a. In EST, the intermediate values of h_1 , h_2 and h_3 are evaluated using a two-dimensional linear interpolation algorithm from the tabulated values.

One modification of the theory was incorporated into the EST computer program to change the level of the theoretical limit load so that it more closely approximated the observed experimental results. The change involved utilizing a limit load (P_O) calculated from the product of a constant k (dependent on material and geometry) and the theoretical limit load, i.e.

$$P_{O} = kP_{O}^{T}$$
 (20)

in all those equations which utilized the limit load in the calculation of the elastic-plastic parameters. The value of k is an input to the computer program and would typically be set equal to 1 for the first set of runs. Subsequent runs made with this interactive program would then be made to bring the theoretical load-displacement behavior more in line with the observed behavior.

3.2 OVERALL FLOW DIAGRAM

Flow diagrams of the computer program are shown in Figures 10 and 11. The call to the subroutine RDDATA will initiate the entering of the test conditions interactively. Tabulated values of h_1 , h_2 and h_3 in Tables 2 through 6 are organized into separate subroutines in the program. Once the geometry and stress condition are interactively chosen, the program calls the corresponding subroutine with h_1 , h_2 and h_3 .

With the input material constants and crack length information, the program makes a call to subroutine TBL2. Subroutine TBL2 is a module which returns the linearly interpolated values of h_1 , h_2 and h_3 to the program for the given n and a/b condition.

After the interpolation of the corresponding h_1 , h_2 and h_3 , the program will make calls to the calculating subroutines JKCAL, LDSCAL and CODCAL. The subroutine JKCAL is written in such a way that for the selected geometry, it will compute K, P_0^T , γ_y and, elastic and plastic components of the

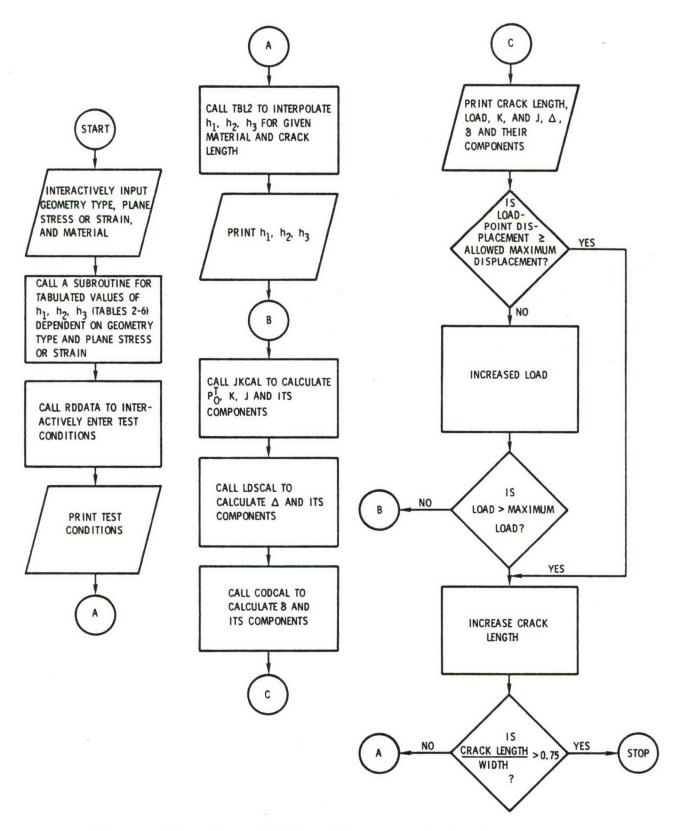


Figure 10. Overall Flow Diagram of the Computer Program.

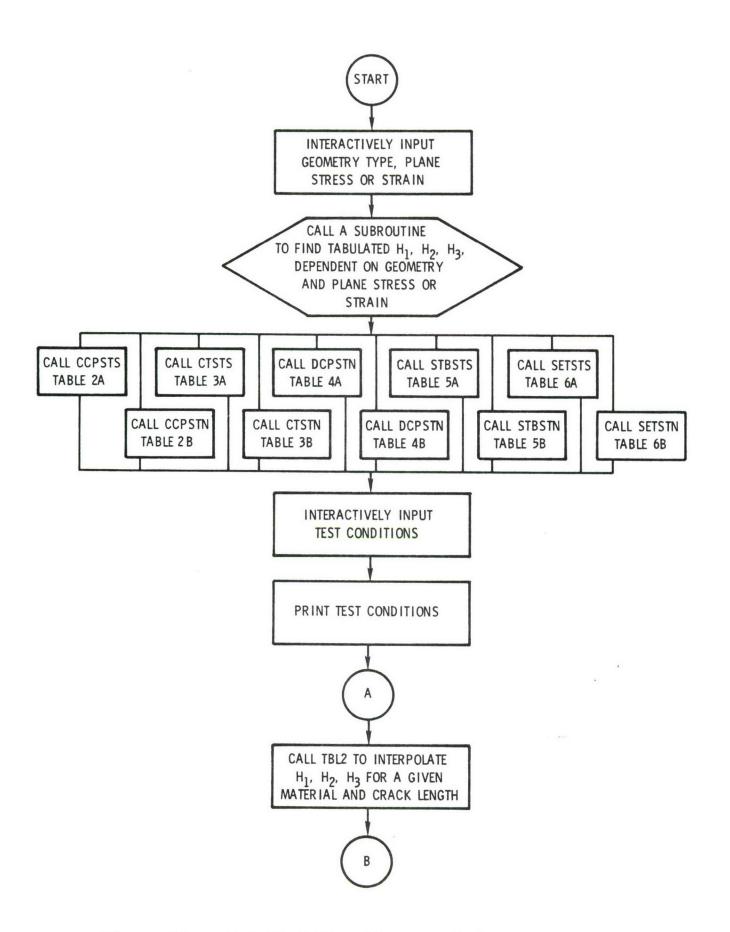


Figure 11a. Detailed Flow Diagram of the Computer Program.

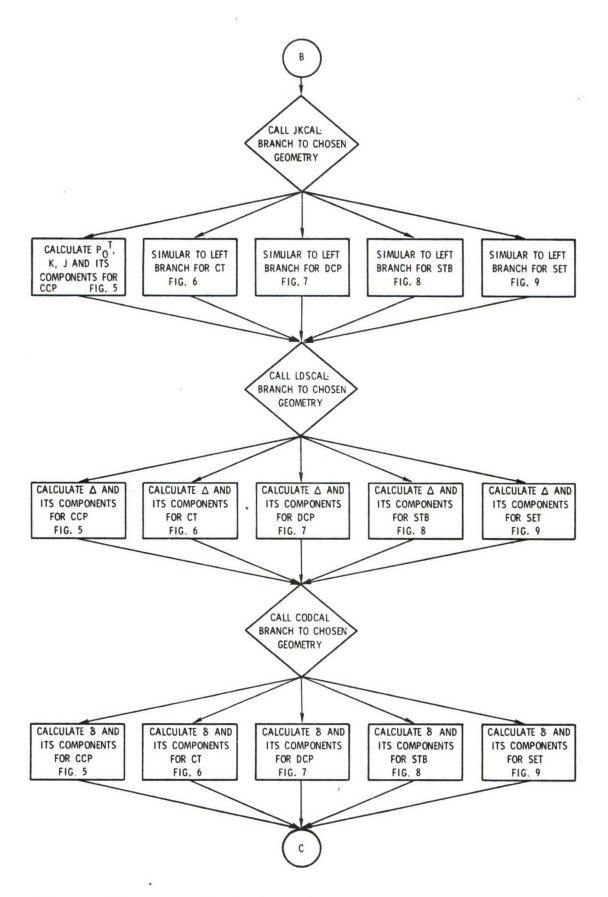


Figure 11b. Detailed Flow Diagram of the Computer Program.

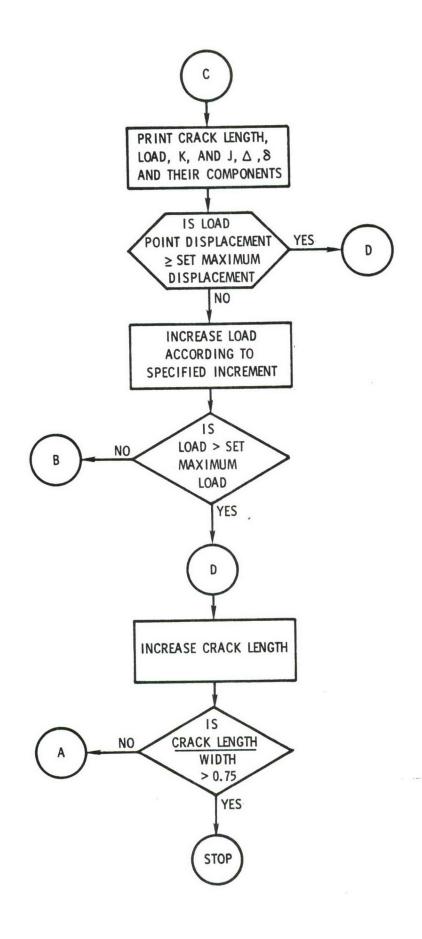


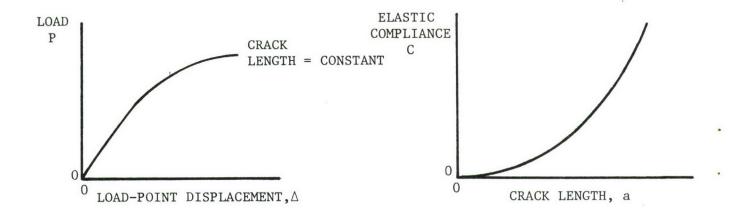
Figure 11c. Detailed Flow Diagram of the Computer Program.

J-integral and return these values to the main program. Subroutines LDSCAL and CODCAL compute and return the values of Δ_n^e , Δ_c^e and, δ^e and δ^ρ respectively, to the main program.

Once these calculations are completed for the given crack length and load, calculated values are printed. Then the load values and crack length values are increased and calculations are performed as described in the flow diagrams in Figures 10 and 11. This procedure continues until a/b exceeds 0.75.

3.3 PROGRAM OUTPUT

The program has been organized so that the output is currently restricted to yield information on only one geometry and one material for each run. The major variable controlling output is physical crack length; the other parameters are described as a function of increasing load for each crack length. Table 7 has been prepared to provide a listing of the output parameter symbols and their description. Appendix B provides the output for a typical run. Table 8 describes some of the cross-correlations which are possible based on the EST program output. Figure 12 was prepared to schematically describe how several of the possible cross-correlations would appear. Alternate presentation schemes are possible through a slight reconfiguration of the algorithm described in the flow diagrams.



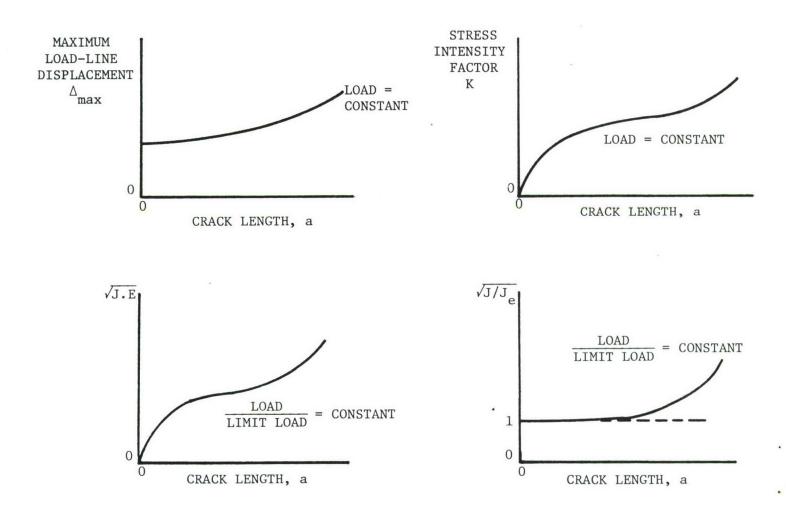


Figure 12. Sample of Program Output Presented in Schematic Diagrams.

TABLE 7
PARAMETER OUTPUT AND THEIR SYMBOLS

Computer Symbol	Text Symbol	Description	Units (English/Metric)
A	a	Physical Crack Length	in/m
AE	a _e	Effective Crack Length	in/m
ALPHA	α	Hardening Constant	
CMOD	δ	Crack Mouth Opening Displacement	in/m
Е	E	Elastic (or Young's) Modulus	ksi/Pa
EC	$\Delta_{\mathbf{C}}^{\mathbf{e}}$	Elastic Crack (Load- line) Displacement	in/m
EN	$\Delta_{\mathbf{n}}^{\mathbf{e}}$	Elastic No-Crack (Load-line)Displace- ment Component	in/m
EPO	εο	Reference Strain	
Hl	h ₁ (a/b,n)	J-Integral Plastic Function	
Н2	$h_2(a/b,n)$	CMOD Plastic Function	
н3	h ₃ (a/b,n)	Plastic Load-line Displacement Function	
J	J	J-Integral	kip-in/N-m
K	K or SIF	Stress Intensity Factor	ksi√in/Pa√m
LC	k	Limit Load Correction Factor	,

TABLE 7 (Continued)
PARAMETER OUTPUT AND THEIR SYMBOLS

Computer Symbol	Text Symbol	Description	Units (English/Metric)
			*
LDISP	Δ	Load Point Displacement	in/m
N	n	Ramberg-Osgood Exponent	
PC	$\Delta_{\mathbf{p}}^{\mathbf{c}}$	Plastic Crack (Load Line) Displacement	in/m
PN	Δ_n^p	Plastic No-Crack (Load Line)	in/m
P/PO	P/P _O	Ratio of Load to Limit Load	
PZC		<pre>Plastic Zone Correction Factor (Monotonic=1; Cyclic=2)</pre>	
SIGO	σ_{o}	Yield Strength	ksi/Pa
SPAN	2L	Gauge Length	in/m
SQRT(EJ)	√EJ	Square Root of (E.J)	ksi√in/Pa√m
SQRT(J/JE)	√J/Je	Square Root of (J/J^e)	
TH		Thickness of the Speçimen	in/m

TABLE 8

POTENTIAL CROSS-CORRELATIONS POSSIBLE WITH PROGRAM OUTPUT

Primary Independent Variable (Abcissa)

Dependent Variable (Ordinate) Secondary Independent Variable (Multiple Curves)

BASIC EXPERIMENTAL PARAMETERS

Load-line Displacement Load

Crack Length

- Total
- Elastic Component
- Plastic Component

Crack Length

Elastic Compliance

Maximum Load-line Displacement

Load

Load to Limit Load

Ratio

Load

Maximum Crack Mouth

Opening Displacement

Load

FATIGUE AND FRACTURE RELATED PARAMETERS

Crack length

Plastic Zone Size

Load, Field Parameters

Effective Crack Length

Load, Field

Parameters

Stress Intensity Factor

J-Integral (HRR Field)

Load, Displacement,

• √J.E

Load to Limit Load Ratio

Load

• $\sqrt{J/J_e}$

Crack Tip Opening Displacement

Load

Load-Line Displacement J-Integral (HRR Field)

Crack Length

• √J.E

• $\sqrt{J/J_e}$

3.4 PROGRAM INPUT REQUIREMENTS

Necessary inputs to the EST computer program are material, structural geometry, and loading properties. A summary listing of the input parameters and their symbols are presented in Table 9 and a sample input listing is provided in Appendix C. As can be noted from this listing, the material hardening parameter α does not appear. The reason for this is that EST calculates α (ALPHA) internally assuming (1) that the constitutive equation is the Ramberg-Osgood stress-strain relationship:

$$\frac{\varepsilon}{\varepsilon_0} = \frac{\sigma}{\sigma_0} + \alpha \left(\frac{\sigma}{\sigma_0}\right)^n \tag{21}$$

(2) that the reference stresses and strains are related by σ_{0} = $E\epsilon_{0}$, (3) that

$$\alpha \varepsilon_{O} = \alpha \left(\frac{\sigma_{O}}{E} \right) = 0.002 \tag{22}$$

and (4) that the hardening exponent n is obtained by least squares fitting procedures. Equation 22 was derived with the use of Figure 13 and an analysis of Equation 21 for the condition $\sigma = \sigma_0$ which shows

$$\varepsilon_{\mathbf{y}} = \varepsilon \Big|_{\sigma = \sigma_{0}^{c}} = \varepsilon_{0} (1 + \alpha) \tag{23}$$

When the EST program is compiled and run, the program prompts the user for necessary information. Input of the geometry, as well as selection of plane stress or plane strain condition allow the program to select the appropriate table of values for h_1 , h_2 and h_3 functions presented in subsection 2.3. The EST program calculates the EPFM parameters for different loads, starting from the user selected minimum load and ending at the user selected maximum load with load increments selected by the user for a given initial a/b ratio. By a suitable selection of crack length increment, it is possible to calculate the EPFM parameters of different crack lengths starting from the initial (a/b) for different loads.

TABLE 9
EST COMPUTER PROGRAM INPUTS

EST SYMBOL	TEXT SYMBOL	PROPERTY	DEFAULT VALUE	ENGLISH/ METRIC UNITS
SIGO	σo	Material 0.2 Percent Yield Strength		ksi/Pa
N	n	Ramberg-Osgood Exponent		
E	E	Elastic Modulus		ksi/Pa
LC	k	Correction for Limit Load	1.0	
PZC		Plastic Zone Correction Factor	1.0	
		Plane Stress/Plane Strain Condition		
		Specimen Geometry (CCP, CT, DCP, STB, SET)		
W	2b	Structural Width		in/m
	L	Half Span		in/m
TH		Thickness		in/m
A/B	a/b	Initial Crack Length/ Width Ratio	0.25	
		Crack Length Increment		in/m
		Load/Displacement		
PMAX		Maximum Load		kips/N
PMIN		Minimum Load	0.0	kips/N
		Load Increment		kips/N
DISMAX		Maximum Load-Line Displacement		in/m

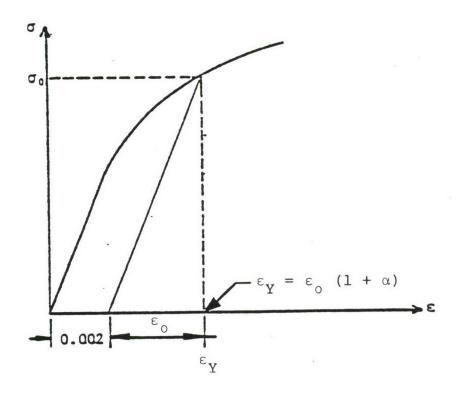


Figure 13. Material Stress-Strain Curve Describing the Values of α and $\epsilon_{_{\hbox{\scriptsize O}}}.$

In the input prompts, the plastic zone correction factor (PC) is taken as either 1 or 2. The value PC is used in Equation 16 as a multiplication factor on the yield strength σ_0 ; when PC = 1 the loading is monotonically icreasing, when PC = 2 the loading is cyclic.

There are several restrictions or bounds on the calculations that result from input or default conditions. The most obvious bounds are the limits on the load or load-line displacement values that are created by input. However, because the tabular values of the h_1 , h_2 , and h_3 functions are limited to the range of a/b between 0.25 and 0.75, the EST program frustrates any attempt to compute the value of these functions outside the allowable a/b range by resetting upper and lower limits. Another default condition occurs whenever the handling exponent (n) exceeds 20 in order to ensure that the h_1 , h_2 , h_3 functions are not extrapolated beyond the tabulated values given in subsection 2.3.

SECTION 4

SUMMARY

A review of the theoretical basis for the estimation scheme proposed by Hutchinson and co-workers^{1,2} and Shih and co-workers^{3,6} was presented in previous sections. A computer program was written to implement the above estimation scheme. Certain modifications were incorporated into the program such that the limit load behavior computed by the program is compatible with the experimentally observed limit load behavior. The computer program can be used to calculate the elastic-plastic behavior of five different specimen geometries. Possible useful outputs from the program are given in Table 8.

REFERENCES

- Goldman, N. L. and Hutchinson, J. W., "Fully Plastic Crack Problems: The Center-Cracked Strip under Plane Strain," Int. J. Solids Structures, 1975, Vol. 11, pp. 575-591.
- Hutchinson, J. W., Needleman, A., and Shih, C. F., "Fully Plastic Crack Problems in Bending and Tension," <u>Fracture</u> <u>Mechanics</u>, ed. N. Perrone, et al., University of Virginia, <u>1978</u>, pp. 515-527.
- 3. Shih, C. F. and Hutchinson, J. S., "Fully Plastic Solutions and Large Scale Yielding Estimates for Plane Stress Crack Programs." J. of Engineering Materials and Technology, 1976, Vol. 98, pp. 289-295.
- 4. Shih, C. F. and Kumar, V., "Estimation Techniques for the Prediction of Elastic-Plastic Fracture of Structural Components of Nuclear Systems," First Semiannual Report, July 1978-January 1979 for EPRI Contract RP 1237-1, General Electric Company, Schenectady, N. Y., June 1, 1979.
- 5. Kumar, V., German, M. D. and Shih, C. F., "Estimation Technique for the Prediction of Elastic-Plastic Fracture of Structural Components of Nuclear Systems," Combined Second and Third Semiannual Report, Feb. 1979 to Jan. 1980 for EPRI, General Electric Company, SRD-80-094.
- 6. Shih, C. F., "J-Integral Estimates for Strain Hardening Materials in Antiplane Shear Using Fully Plastic Solutions,"

 <u>Mechanics of Crack Growth</u>, ASTM Special Technical Publication 590, 1976, pp. 3-22.
- 7. Hutchinson, J. W. "Plastic Stress and Strain Fields at the Crack Tip," <u>Journal of Mechanics and Physics of Solids</u>, 1968, pp. 13-31; pp. 337-347.
- 8. Rice, J. R., and Rosengren, G. F., "Plane Strain Deformation Near a Crack Tip in Power-Law Hardening Material," <u>Journal</u> of Mechanics and Physics and Solids, 1968, pp. 1-12.
- 9. Hult, J. A. H. and McClintock, F. A., "Elastic-Plastic Stress and Strain Distribution Around Sharp Notches Under Repeated Shear," 9th Int. Cong. of Appl. Mechanics., University of Brussels, Vol. 8, 1957, pp. 51-58.
- 10. Koskinen, M. F., "Elastic-Plastic Deformation of a Single Grooved Flat Plate Under Longitudinal Shear," J. of Basic Eng., Trans. ASME, Vol. 86, Series D, 1963, pp. 585-588.

- 11. Paris, P. C., "Fracture Mechanics in the Elastic-Plastic Regime," Flow Growth and Fracture, ASTM STP 631, American Society for Testing and Materials, 1977, pp. 3-27.
- 12. Rice, J. R., "A Path Independent Integral and the Approximate Analysis of Strain Concentration by Notches and Cracks," J. of Appl. Mech., Vol. 35, 1968, pp. 379-386.
- 13. Rice, J. R. and Tracey, D. M., "Computational Fracture Mechanics," Numerical and Computer Methods in Structural Mechanics (Ed. S. J. Fenves et al.), Academic Press, N. Y., 1973, pp. 585-623.
- 14. Shih, C. F., "Relationship Between the J-Integral and the Crack Opening Displacement for Stationary and Extending Cracks," General Electric Co. TIS Report No. 79CRD075, April 1979.
- 15. Kumar, V., Private Communications (July, 1980).
- 16. Bucci, R. J., Paris, P. C., Landes, J. D. and Rice, J. R., "J-Integral Estimation Procedures," Fracture Toughness, ASTM STP514, 1972, pp. 40-69.
- 17. Tada, H., Paris, P. C. and Irwin, G. R., <u>The Stress Analysis of Cracks Handbook</u>, Del Research Corporation, Hellertown, PA, 1973.
- 18. Ilyushin, A. A., "The Theory of Small Elastic-Plastic Deformation," Prikadnaia Matematika i Mekhanika, P.M.M., Vol. 10, 1946, p. 347.
- 19. Edmunds, T. M. and Willis, J. R., "Matched Asymptotic Expansions in Nonlinear Fracture Mechanics I, II and III," J. Mech. Physics of Solids, Vol. 24, 1976, pp. 205 and 225, Vol. 25, 1977, p. 424.

APPENDIX A

A LISTING OF THE FORTRAN IV
PROGRAM EST
DESIGNED TO RUN INTERACTIVELY
ON THE CDC CYBER 175
COMPUTER SYSTEM

```
PROGRAM EST (INPUT, OUTPUT, TAPE6, TAPE1=INPUT)
      COMMON/A/E, EA, PLCOR, SIGO, XLL, B, FACTOR, XN, ALPHA, EPO, TH, PP1, IPL,
     $ A,H1,H2,H3,PP,ZZ,ZZE,PO,AE,XKSQ,XJ,BETA,PI
      COMMON/B/DELTE, DELTP, DELTA
      COMMON/C/DELCE, DELCP, DELNE, DELNP, DELC, DELN, DELT, DELE, DELP
      COMMON/D/X(10),Y(10),Z1(100),Z2(100),Z3(100),IFLAG,METENG
      COMMON/E/ PMIN, PMAX, PIN, ZZO, AIN, DISMAX
      COMMON/F/NUMAB, NUMN, ITYPE
      COMMON/X/PPPO,XJJE
C
C
       PROGRAM EST CALCULATES J, DISPLACEMENT, AND DELTA
C
     FOR THE FOLLOWING SPECIMENS:
C
C
         1) CENTER CRACKED PANEL (CCP) --- PLANE STRAIN OR PLANE STRESS
C
         2) COMPACT TENSION SPECIMEN (CT) --- PLANE STRAIN
C

    DOUBLE-EDGE CRACK PANEL (DCP) --- PLANE STRAIN

C
         4) SINGLE-EDGE CRACK PANEL IN 3-POINT BENDING --- PLANE STRAIN
C
         5) SINGLE-EDGE CRACK PANEL IN TENSION (SET) --- PLANE STRAIN
C
C
         THE VALUES OF DISPLACEMENT VS. J, LOAD, AND DELTA ARE
C
     WRITEN TO A FILE USED TO GENERATE PLOTS (UNIT 7).
C
11
      REWIND 6
      XNU=0.3
      IFLAG=0
      PRINT2100
      FORMAT (*0ENTER SPECIMEN GEOMETRY*,/,
2100
       * (CCP,CT,DCP,STB,SET) == | *)
      READ(1,100)JTYPE
100
      FORMAT (A3)
      PRINT2200
2200
      FORMAT(* PLANE STRESS (0) OR PLANE STRAIN (1) ? *)
      READ*, IPL
      PRINT2500
2500
      FORMAT(* MATERIAL == | *)
      READ(1,2600) MAT
      FORMAT (A10)
2600
       WRITE (6,200) MAT
      FORMAT(* MATERIAL: *,A10)
200
      IF(JTYPE.EQ.3HCCP.AND.IPL.EQ.0)
     $ CALL CCPSTS
      IF (JTYPE.EQ.3HCCP.AND.IPL.EQ.1)
     $ CALL CCPSTN
      IF(JTYPE.EQ.3HCT .AND. IPL.EQ.0)
     $ CALL CTSTS
      IF (JTYPE.EQ.3HCT .AND. IPL.EQ.1)
     $ CALL CTSTN
      IF (JTYPE.EQ.3HDCP .AND. IPL.EQ.0)
     $ CALL DCPSTS
      IF (JTYPE.EQ.3HDCP .AND. IPL.EQ.1)
     $ CALL DCPSTN
      IF (JTYPE.EQ.3HSTB .AND. IPL.EQ.0)
     $ CALL STBSTS
      IF (JTYPE.EQ.3HSTB .AND. IPL.EQ.1)
     $ CALL STBSTN
```

```
IF (JTYPE.EQ.3HSET .AND. IPL.EQ.0)
     $ CALL SETSTS
      IF(JTYPE.EQ.3HSET .AND. IPL.EQ.1)
     $ CALL SETSTN
      IF (IFLAG.EQ.0) PRINT2300
      FORMAT(*0INVALID GEOMETRY*)
      IF(IFLAG.EQ.0) GO TO 11
C
C
      READ VARIABLES INTERACTIVELY
C
      CALL RDDATA
      A0=ZZ0*B
      A = A0
      BETA=2.0
      IF(IPL .NE. 0) BETA=6.0
      WRITE (6,300)
300
      FORMAT(6X,*TH*,9X,*SPAN*,11X,*N*,13X,*SIGO*,7X,*PZC*,14X,
            *E*,8X,*WIDTH*,10X,*LC*,9X,*ALPHA*,10X,*EPO*,/)
      EA=E
      IF(IPL.NE.0)EA=E/(1.-XNU**2)
      EP0=SIG0/E
      ALPHA=0.002/EP0
      SPAN=2*XLL
      B2=B
      IF(ITYPE.EQ.1.OR.ITYPE.EQ.3) B2=B*2
      WRITE (6,400) TH, SPAN, XN, SIGO, PLCOR, E, B2, FACTOR, ALPHA, EPO
400
      FORMAT(10G13.4)
12
      ZZ=A/B
      IF(XN.GT.20.0) XN=20.0
      WRITE (6,1300) XN, ZZ
1300
      FORMAT(*0 X = *,G14.4,5X,*A/B = *,G14.4)
C
C
        INTERPOLATE FOR VALUES OF H1, H2, H3
C
      CALL TBL2(X,Y,Z1,NUMN,NUMAB,NUMN,XN,ZZ,H1,IER)
      CALL TBL2(X,Y,Z2,NUMN,NUMAB,NUMN,XN,ZZ,H2,IER)
      CALL TBL2(X,Y,Z3,NUMN,NUMAB,NUMN,XN,ZZ,H3,IER)
      WRITE (6,1400) H1, H2, H3
                H1 = *,F12.4,3X,*H2 = *,F12.4,3X,*H3 = *,F12.4)
1400
      FORMAT (*
      IF (METENG .EQ. 1HM) WRITE (6,1500)
      IF (METENG .EQ. 1HE) WRITE (6,1600)
      FORMAT(*0ALL RESULTS IN METRIC UNITS*,/)
1500
1600
      FORMAT(*OALL RESULTS IN ENGLISH UNITS*,/)
      IF (ITYPE.EQ.1) WRITE (6,500)
      FORMAT (T4, *2A*, T8, *2A/W*, T15, *2AE*, T21, *P/PO*, T31,
500
     $ *LOAD*,T42,*K*,T53,*J*,T59,*SQRT(EJ)*,T68,*SQRT(J/JE)*,
     $ T81,*CMOD*,T91,*LDISP*,T102,*EN*,T111,*PN*,T119,*EC*,T128,*PC*)
      IF(ITYPE.NE.1) WRITE(6,600)
600
      FORMAT(T5,*A*,T9,*A/W*,T15,*AE*,T21,*P/P0*,T31,
     $ *LOAD*,T42,*K*,T53,*J*,T59,*SQRT(EJ)*,T68,*SQRT(J/JE)*,
     $ T81,*CMOD*,T91,*LDISP*,T102,*EN*,T111,*PN*,T119,*EC*,T128,*PC*)
      PP1=PMIN
      CONTINUE
13
```

```
C
C
            CALCULATE J AND K
      CALL JKCAL
C
            CALCULATE LOAD-POINT DISPLACEMENT
C
      CALL LDSCAL
C
            CALCULATE CRACK-OPENING DISPLACEMENT
C
      CALL CODCAL
C
      XK=SQRT(XKSQ)
      XXJ=SQRT(XJ*EA)
      XJJE=SQRT(XJJE)
      AE2=AE
      A2=A
      IF(ITYPE .EQ. 1) A2=A*2
      IF(ITYPE .EQ. 1) AE2=AE*2
      WRITE(6,700)A2,ZZ,AE2,PPP0,PP1,XK,XJ,XXJ,XJJE,DELTA,DELT,DELNE,
     $DELNP, DELCE, DELCP
      FORMAT(1H ,2F5.3,F6.3,E10.3,F9.3,3E10.3,F10.7,E10.3,E9.3,E10.3,
700
     $ 3E9.3)
      IF (DELT .GE. DISMAX) GO TO 15
      PP1=PP1+PIN
      IF (PP1 .LT. PMAX) GO TO 13
15
      CONTINUE
      A=A+AIN
      IF (A/B .LE. 3./4.) GO TO 12
      STOP
      END
C
```

READ(1,2000)KANS

IF (KANS.NE.1HY) READ*, DISMAX

INITIALIZES VARIABLES AND PROMPTS FOR USER INPUT ** COMMON/A/E, EA, PLCOR, SIGO, XLL, B, FACTOR, XN, ALPHA, EPO, TH, PP1, IPL, \$A,H1,H2,H3,PP,ZZ,ZZE,PO,AE,XKSQ,XJ,BETA,PI COMMON/D/X(10), Y(10), Z1(100), Z2(100), Z3(100), IFLAG, METENGCOMMON/F/NUMAB, NUMN, ITYPE COMMON/E/ PMIN, PMAX, PIN, ZZO, AIN, DISMAX DATA PMIN/0.0/,PIN/.25/,PMAX/5.3/,ZZO/.25/,XN/36./, \$ SIGO/33E3/,PLCOR/1./,E/17E6/,XLL/2./,B/1.995/, \$ TH/0.498/,AIN/0.1/,FACTOR/1./,DISMAX/0.01/ PRINT1600 READ(1,2000) METENG PRINT100, PMIN READ(1,2000)KANS IF (KANS.NE.1HY) READ*, PMIN PRINT200, PIN READ(1,2000)KANS IF (KANS.NE.1HY) READ*, PIN PRINT300, PMAX READ(1,2000) KANS IF (KANS.NE.1HY) READ*, PMAX PRINT500,ZZO READ(1,2000)KANS IF (KANS.NE.1HY) READ*, ZZO PRINT600, XN READ(1,2000)KANS IF (KANS.NE.1HY) READ*,XN PRINT700,SIGO READ(1,2000)KANS IF (KANS.NE.1HY) READ*, SIGO PRINT800, PLCOR READ(1,2000)KANS IF (KANS.NE.1HY) READ*, PLCOR PRINT900,E READ(1,2000)KANS IF (KANS.NE.1HY) READ*, E PRINT1000,XLL READ (1,2000) KANS IF (KANS.NE.1HY) READ*,XLL PRINT1100,B READ(1,2000)KANS IF (KANS.NE.1HY) READ*, B IF(ITYPE .EQ. 1 .OR. ITYPE .EQ. 3)B=B/2.0PRINT1200,TH READ(1,2000)KANS IF (KANS.NE.1HY) READ*, TH PRINT1300, AIN READ(1,2000)KANS IF (KANS.NE.1HY) READ*, AIN PRINT1400, FACTOR READ(1,2000)KANS IF (KANS.NE.1HY) READ*, FACTOR PRINT1500, DISMAX

```
FORMAT (*01F THE FOLLOWING VALUES ARE CORRECT, *,/,
100
     $* RESPOND WITH A "Y". IF THEY ARE INCORRECT, *,/,
     $* ANSWER "N", THEN ENTER THE CORRECT VALUE*,/,
     \$*OMINIMUM LOAD (KIPS/N) = *,G10.5,* (Y/N)? *)
      FORMAT(* LOAD INCREMENT (KIPS/N) = *,Gll.5,* (Y/N)? *)
200
      FORMAT(* MAXIMUM LOAD(KIPS/N) = *, G7.2, * (Y/N)? *)
300
      FORMAT(* INITIAL A/B = *,Gll.5,* (Y/N)? *)
500
      FORMAT(* RAMBERG-OSGOOD EXPONENT - N = *,Gll.5,* (Y/N)? *)
600
      FORMAT(* YIELD STRENGTH - SIGO (KSI/PA) = *,G14.5,* (Y/N)? *)
700
      FORMAT(* PLASTIC ZONE CORRECTION FACTOR - PZC= *,G14.5,* (Y/N)? *)
800
      FORMAT(* ELASTIC MODULUS - E (KSI/PA) = *,G14.5,* (Y/N)? *)
900
      FORMAT(* HALF SPAN (IN/M) = *,Gll.5,* (Y/N)? *)
1000
      FORMAT (* WIDTH - W (IN/M) = *,Gll.5,* (Y/N)? *)
1100
      FORMAT(* THICKNESS - TH (IN/M) = *,G12.5,* (Y/N)? *)
1200
      FORMAT(* CRACK LENGTH INCREMENT (IN/M) = *,G10.5,* (Y/N)? *)
1300
      FORMAT(* CORR. FACTOR FOR LIMIT LOAD - LC = *,G10.5,* (Y/N)? *)
1400
      FORMAT(* MAXIMUM DISPLACEMENT (IN/M) = *,Gl0.5,* (Y/N)? *)
1500
      FORMAT(* E(NGLISH) OR M(ETRIC) ? *)
1600
      FORMAT(A1)
2000
      RETURN
      END
```

SUBROUTINE TBL2 (X, Y, Z, NX, NY, NDX, XO, YO, ZO, IER)

TBL2 FROM THE COMPUTER CENTER LIBRARY OF 6600 ROUTINES

SUBROUTINE TBL2

PERFORMS DOUBLE LINEAR INTERPOLATION FROM A TABLE OF VALUES OF Z VERSES X AND Y.

USAGE

CALL TBL2 (X,Y,Z,NX,NDX,XO,YO,ZO,IER)

DESCRIPTION OF PARAMETERS

- X VALUES OF THE FIRST INDEPENDENT VARIABLE IN INCREASING ORDER
- Y VALUES OF THE SECOND INDEPENDENT VARIABLE IN INCREASING ORDER
- Z VALUES OF THE DEPENDENT VARIABLE. Z(I,J) IS THE VALUE OF Z CORRESPONDING TO X(I), Y(J) AND IS REFERENCED AS Z((J-1)*NDX+I)
- NX NUMBER OF ENTRIES IN THE X ARRAY
- NY NUMBER OF ENTRIES IN THE Y ARRAY
- NDX DIMENSION IN THE X DIRECTION OF THE Z ARRAY (NDX.GE.NX)
- XO X COORDINATE OF POINT AT WHICH INTERPOLATION IS DESIRED
- YO Y COORDINATE OF POINT AT WHICH INTERPOLATION IS DESIRED
- ZO THE COMPUTED INTERPOLATED VALUE
- IER = 0 INTERPOLATION SUCESSFULLY PERFORMED
 - = 1 EXTRAPOLATION SUCESSFULLY PERFORMED
 - = 2 ERROR CONDITION. EITHER TWO ADJACENT INDEPENDENT VARIABLES ARE NOT IN INCREASING ORDER OR NX IS GREATER THAN NDX

METHOD

XO AND YO ARE LOCATED RELATIVE TO THE X AND Y ARRAYS. IF EXTRAPOLATION IS NOT NECESSARY, REPEATED BISECTION OF THE INDICES OF THE LISTS OF X AND Y VALUES IS CARRIED OUT UNTIL BOTH ARGUMENTS ARE ISOLATED BETWEEN PAIRS OF CONSECUTIVE VALUES. AFTER LOCATION OF THE NEAREST PAIR OF ARGUMENTS FOR EACH OF THE TWO INDEPENDENT VARIABLES, DOUBLE LINEAR INTERPOLATION (EXTRAPOLATION) IS PERFORMED.

EXAMPLE - GIVEN THE FUNCTION Z = X+2Y, X=1,2 AND Y=0,1 STORAGE AS A ONE DIMENSIONAL OR TWO DIMENSIONAL ARRAY COULD BE AS

FOLLOWS Z(1) = Z(1,1) = 1 Z(3) = Z(1,2) = 3

Z(2) = Z(2,1) = 2 Z(4) = Z(2,2) = 4WHERE THE DIMENSION OF Z IS Z(4) OR Z(2,2) AND NDX = 2

OR Z(1) = Z(1,1) = 1 Z(5) = Z(1,2) = 3

Z(2) = Z(2,1) = 2 Z(6) = Z(2,2) = 4

WHERE THE DIMENSION OF Z IS Z(16) OR Z(4,4) AND NDX = 4.

DIMENSION X(1), Y(1), Z(1)

IF (NX.GT.NDX) GO TO 12

```
C
      DETERMINE WHETHER EXTRAPOLATION IS NECESSARY IN THE X DIRECTION
      I1 = 1
      IF (XO.LT.X(1)) GO TO 4
      IF (XO.GT.X(NX)) GO TO 3
C
C
      LOCATE XO WITHIN X(I) ARRAY FOR INTERPOLATION
      IER = 0
      I2 = NX
    1 IF ((I2-I1).LT.2) GO TO 5
      I = (I1+I2)/2
      IF (XO.LT.X(I)) GO TO 2
      I1 = I
      GO TO 1
    2 I2 = I
      GO TO 1
C
C
      SET INDICES AND FLAG FOR EXTRAPOLATION
    3 I1 = NX-1
    4 IER = 1
C
C
      DETERMINE WHETHER EXTRAPOLATION IS NECESSARY IN THE Y DIRECTION
    5 J1 = 1
      IF (YO.LT.Y(1)) GO TO 9
      IF (YO.GT.Y(NY)) GO TO 8
C
      LOCATE YO WITHIN Y(I) ARRAY FOR INTERPOLATION
      J2 = NY
    6 IF ((J2-J1).LT.2) GO TO 10
      J = (J1+J2)/2
      IF (YO.LT.Y(J)) GO TO 7
      J1 = J
      GO TO 6
    7 J2 = J
      GO TO 6
C
C
      SET INDICES AND FLAG FOR EXTRAPOLATION
    8 Jl = NY-1
    9 IER = 1
C
      COMPUTE ZO USING BIVARIATE INTERPOLATION
   10 DIV = (X(I1+1) - X(I1)) * (Y(J1+1) - Y(J1))
      IF (DIV) 12,12,11
   11 I11 = (J1-1)*NDX + I1
      I12 = I11 + NDX
      X20 = X(I1+1) - XO
      X01 = XO - X(I1)
C
      ZO = (Y2-Y0)((X2-X0)Z11 + (X0-X1)Z21) +
C
           (YO-Y1)((X2-XO)Z12 + (XO-X1)Z22) / (X2-X1)(Y2-Y1)
      ZO = ((Y(J1+1)-YO)*(X20*Z(I11) + X01*Z(I11+1)) +
            (YO-Y(J1))*(X20*Z(I12) + X01*Z(I12+1))) /DIV
      RETURN
   12 IER = 2
      RETURN
      END
```

```
C
CC
           CALCULATES ELASTIC AND PLASTIC COMPONENTS OF J **
      COMMON/A/E, EA, PLCOR, SIGO, XLL, B, FACTOR, XN, ALPHA, EPO, TH, PP1, IPL,
     $ A,H1,H2,H3,PP,ZZ,ZZE,PO,AE,XKSQ,XJ,BETA,PI
      COMMON/X/PPPO,XJJE
      COMMON/F/NUMAB, NUMN, ITYPE
      FSQCCP(Z) = 1./(COS((3.14159/2.)*Z))
      FSOCT(Z) = (((2+Z)*(0.886+4.64*Z-13.32*Z**2+14.72*Z**3
     $-5.6*2**4)
      FSQSTB(Z) = (1.09-1.735*Z+8.2*Z**2-14.18*Z**3+14.57*Z**4)**2
      FSQDCP(Z) = (1.122-0.561*Z-0.205*Z**2+0.471*Z**3-0.19*Z**4)**2/(1-Z)
      FSQST(Z) = (2/(PI*Z))*TAN(PI*Z/2.)*((0.752+2.02*Z+0.37)
     $ *(1-SIN(PI*Z/2.))**3)/COS(PI*Z/2.))**2
      PI=3.14159
      PP=PP1/TH
      ZZ=A/B
      GO TO (1,2,3,4,5) ITYPE
C
         CCP SPECIMEN
 1
      P0=2.*SIG0*(B-A)*FACTOR
      IF(IPL .NE. 0) P0=2.31*SIG0*(B-A)*FACTOR
      XKSQ = (PP**2*PI*A*FSOCCP(ZZ))/(4.*B**2)
      PHI=1./(1.+(PP/P0)**2)
      RY = (XN-1.) *XKSQ/(BETA*PI*(XN+1.)*(SIGO*PLCOR)**2)
      AE=A+PHI*RY
      ZZE=AE/B
      XJE=(PI*AE*FSOCCP(ZZE)*(PP**2))
            /(EA*4.*B**2)
      XJP=ALPHA*SIG0*EP0*A*((B-A)/B)*H1*(PP/P0)**(XN+1.)
      GO TO 6
        CT SPECIMEN
C
2
      XNETA = SQRT(((2.*A)/(B-A))**2+((4.*A)/(B-A))
          +2.) - ((2.*A)/(B-A)+1.)
      P0=1.071*XNETA*(B-A)*SIGO*FACTOR
      IF(IPL .NE. 0) P0=1.455*XNETA*(B-A)*SIGO*FACTOR
      XKSO = (PP**2*A*FSOCT(ZZ))/(B**2)
      PHI=1./(1.+(PP/P0)**2)
      RY = (XN-1.)*XKSO/(BETA*PI*(XN+1.)*(SIGO*PLCOR)**2)
      AE=A+PHI*RY
      ZZE=AE/B
      XJE = (AE*FSQCT(ZZE)*(PP**2))/(EA*B**2)
      XJP=ALPHA*SIGO*EPO*(B-A)*H1*(PP/PO)**(XN+1)
      GO TO 6
        DCP SPECIMEN
      P0=2.31*SIG0*(B-A)*FACTOR
      IF(IPL.NE.0) P0=5.94*SIGO*(B-A)*FACTOR
      XKSQ = (PP**2*PI*A*FSQDCP(ZZ))/(2.*B)**2
      PHI=1./(1.+(PP/P0)**2)
      RY = (XN-1.)*XKSQ/(BETA*PI*(XN+1.)*(SIGO*PLCOR)**2)
      AE=A+PHI*RY
      ZZE=AE/B
      XJE = (PI*AE*FSQDCP(ZZE)*(PP**2))/(EA*B**2)
      XJP=ALPHA*SIGO*EPO*(B-A)*H1*(PP/PO)**(XN+1.)
      GO TO 6
```

```
STB SPECIMEN
C
      P0=(0.536*SIG0*(B-A)**2/XLL)*FACTOR
      IF(IPL.NE.0) P0=(0.728*SIG0*(B-A)**2/XLL)*FACTOR
      XKSQ=9.*PP**2*XLL**2*PI*A*FSQSTB(ZZ)/((B)**4)
      PHI=1./(1.+(PP/P0)**2)
      RY=(XN-1.) *XKSQ/(BETA*PI*(XN+1.) *(SIGO*PLCOR) **2)
      AE=A+PHI*RY
      ZZE=AE/B
      XJE=(9.*PI*AE*XLL**2*FSQSTB(ZZE)*PP**2)/(EA*(B)**4)
      XJP=ALPHA*SIG0*EP0*(B-A)*H1*(PP/P0)**(XN+1.)
      GO TO 6
        SET SPECIMEN
C
      XNETA = SQRT(1 + (A/(B-A)) **2) - (A/(B-A))
5
      PO=1.455*XNETA*(B-A)*SIGO*FACTOR
      IF(IPL.EQ.0)P0=1.072*XNETA*(B-A)*SIGO*FACTOR
      XKSQ = (PP**2*PI*A*FSQST(ZZ))/(B**2)
      PHI=1.0/(1.0+(PP/P0)**2)
      RY=(XN-1.0)*XKSQ/(BETA*PI*(XN+1.0)*(SIGO*PLCOR)**2)
      AE=A+PHI*RY
      ZZE=AE/B
      XJE = (PI*AE*FSQST(ZZE)*(PP**2))/(EA*B**2)
      XJP=ALPHA*SIGO*EPO*A*((B-A)/B)*H1*(PP/PO)**(XN+1.)
      XJ = XJE + XJP
6
      XJEE = (XKSQ/EA)
      IF(XJEE.EQ.0.0) XJJE=0.0
      IF(XJEE.NE.0.0) XJJE=(XJ/XJEE)
      PPP0=PP/P0
      RETURN
      END
```

```
SUBROUTINE LDSCAL
      COMMON/A/E, EA, PLCOR, SIGO, XLL, B, FACTOR, XN, ALPHA, EPO, TH, PP1, IPL,
     $ A,H1,H2,H3,PP,ZZ,ZZE,PO,AE,XKSQ,XJ,BETA,PI
      COMMON/C/DELCE, DELCP, DELNE, DELNP, DELC, DELN, DELT, DELE, DELP
      COMMON/F/NUMAB, NUMN, ITYPE
C
C
           LOAD POINT DISP CALC. FOR ALL SPECIMENS
C
        DELCE = ELASTIC CRACKED COMPONENT
CC
        DELCP = PLASTIC CRACKED COMPONENT
        DELNE = ELASTIC NO CRACKED COMPONENT
C
        DELNP = PLASTIC NO CRACKED COMPONENT
      V2CCP(Z) = -1.071 + 0.25 \times Z - 0.357 \times Z \times Z + 0.121 \times Z \times Z
             -0.047*Z**4+0.008*Z**5-1.071*(1./Z)*(ALOG(1.-Z))
      V2CT(Z) = (11.062*Z**3-27.209*Z**2+27.977*Z+0.995) /
     ((1-Z)**2)
      V2DCP(Z) = (2/(PI*Z))*(0.0629-0.061*COS(PI*Z/2.)**4
     $ -0.0019*COS(PI*Z/2.)**8+ALOG(1/(COS(PI*Z/2.))))
      V2STB(Z) = (Z/(1-Z))**2*(5.58-19.57*Z+36.82*Z**2
     $ -34.94*Z**3+12.77*Z**4)
      V2ST(Z) = (Z/(1-Z)**2)*(0.99-Z*(1-Z)*(1.3-1.2*Z+0.7*Z**2))
      xnu=0.3
      DELCP=ALPHA*EPO*A*H3*(PP/PO)**XN
      GO TO (1,2,3,4,5) ITYPE
C
         CCP SPECIMEN
1
      DELCE = (2.*AE*V2CCP(ZZE)*PP)/(B*EA)
      DELNE=(PP*XLL)/(B*E)
      DELNP=1.7321*ALPHA*EPO*XLL*((0.433*PP)/(B*SIGO))
     $
            **XN
      GO TO 6
        CT SPECIMEN
C
      DELCE=V2CT(ZZE) *PP/EA
      DELNE=0.0
      DELNP=0.0
      GO TO 6
        DCP SPECIMEN
      DELCE=2*AE*V2DCP(ZZE)*PP/(B*EA)
      DELCP=ALPHA*EPO*(B-A)*H3*(PP/PO)**XN
      DELNE = (PP*XLL) / (B*E)
      DELNP=1.7321*ALPHA*EPO*XLL*((0.433*PP)/(B*SIGO))
      GO TO 6
        STB SPECIMEN
      DELCE = (6*XLL**2*V2STB(ZZE)*PP)/(EA*B**2)
      DELCP=ALPHA*EPO*A*H3*(PP/PO)**XN
      DELNE=((1.-XNU**2)*PP*XLL**3)/(E*0.5*B**3)+((PP*XLL)/(B*E))
        *(1.5*(1.+XNU)-0.3*(1.-XNU**2)-0.75*XNU*(1.-XNU**2))
        -(0.21*(1.-XNU**2)*PP)/E
      DELNP=0.
      GO TO 6
        SET SPECIMEN
      DELCE=(4.*AE*V2ST(ZZE)*PP)/(EA*B)
      DELCP=ALPHA*EPO*A*H3*(PP/PO)**XN
      DELNE=(2.*PP*XLL)/(B*E)
      DELNP=1.7321*ALPHA*EP0*XLL*((0.866*PP)/(2.*B*SIG0))**XN
C
        DELC = TOTAL DISPLACEMENT DUE TO CRACK
        DELN = TOTAL DISPLACEMENT DUE TO NO CRACK
      DELC=DELCE+DELCP
      DELN=DELNE+DELNP
      DELT=DELC+DELN
      DELE=DELNE+DELCE
      DELP=DELNP+DELCP
                                      59
      RETURN
```

END

SUBROUTINE CODCAL

```
C
C
     ** CALCULATES CRACK-OPENING DISPLACEMENT FOR ALL SPECIMENS
         DELTE = ELASTIC COMPONENT
C
         DELTP = PLASTIC COMPONENT
C
      COMMON/A/E, EA, PLCOR, SIGO, XLL, B, FACTOR, XN, ALPHA, EPO, TH, PP1, IPL,
     $ A,H1,H2,H3,PP,ZZ,ZZE,PO,AE,XKSQ,XJ,BETA,PI
      COMMON/B/DELTE, DELTP, DELTA
      COMMON/C/DELCE, DELCP, DELNE, DELNP, DELC, DELN, DELT, DELE, DELP
      COMMON/F/NUMAB, NUMN, ITYPE
      V1CCP(Z) = -0.071 - 0.535 \times Z + 0.169 \times Z \times 2 + 0.02 \times Z \times 3
             -1.071*(1./2)*(ALOG(1.-2))
      V1CT(Z) = (57.694*Z**3-83.166*Z**2+43.315*Z+5.435) /
     \$((1-Z)**2)
      VLDCP(Z) = (2/(PI*Z))*((0.459*SIN(PI*Z/2.))-(0.065*(SIN(PI*Z/2.)))
     $ **3)-(0.007*(SIN(PI*Z/2.))**5)+(ALOG((1/(COS(PI*Z/2.)))
     $ +SORT((1/(COS(PI*Z/2.)))**2-1.)))
      V1STB(Z) = 0.76 - 2.28 \times Z + 3.87 \times Z \times 2 - 2.04 \times Z \times 3 + 0.66 / (1-Z) \times 2
      VIST(Z) = (1.46+3.42*(1-COS(PI*Z/2.)))/COS(PI*Z/2.)**2
      GO TO (1,2,3,4,5) ITYPE
          CCP SPECIMEN
C
      DELTE=(2.*AE*V1CCP(ZZE)*PP)/(B*EA)
 1
      DELTP=ALPHA*EPO*A*H2*(PP/PO)**XN
      GO TO 6
         CT SPECIMEN
      DELTE=V1CT(ZZE)*PP/EA
      DELTP=ALPHA*EPO*A*H2*(PP/PO)**XN
      GO TO 6
         DCP SPECIMEN
C
3
      DELTE=2.*AE*V1DCP(ZZE)*PP/(B*EA)
      DELTP=ALPHA*EP0*(B-A)*H2*(PP/P0)**XN
      GO TO 6
C
         STB SPECIMEN
      DELTE=12.*AE*XLL*V1STB(ZZE)*PP/(B**2*EA)
      DELTP=ALPHA*EPO*A*H2*(PP/PO)**XN
      GO TO 6
C
         SET SPECIMEN
5
      DELTE=4.*AE*V1ST(ZZE)*PP/(B*EA)
      DELTP=ALPHA*EPO*A*H2*(PP/PO)**XN
      DELTA=DELTE+DELTP
6
      RETURN
      END
```

```
SUBROUTINE CCPSTS
     COMMON/D/X(10),Y(10),Z1(100),Z2(100),Z3(100),IFLAG,METENG
     COMMON/F/NUMAB, NUMN, ITYPE
      DATA X/1.,1.5,2.,3.,5.,7.,10.,13.,16.,20./
      DATA Y/0.125,0.2,0.25,0.375,0.5,0.625,0.75,3*0.0/
      DATA Z1/
     $ 2.80, 3.23, 3.54, 4.00, 4.52, 4.76, 4.86, 3.25, 3.95, 3.65,
     $ 2.63, 3.00, 3.15, 3.42, 3.62, 3.65, 3.43, 2.95, 2.89, 2.55,
     $ 2.54, 2.82, 2.97, 3.14, 3.19, 3.11, 2.90, 2.65, 2.47, 2.20,
     $ 2.34, 2.40, 2.53, 2.52, 2.35, 2.17, 1.95, 1.77, 1.61, 1.43,
     $ 2.21, 2.24, 2.20, 2.06, 1.81, 1.63, 1.43, 1.30, 1.17, 1.00,
     $ 2.11, 2.07, 1.91, 1.69, 1.41, 1.22, 1.01, 0.85, 0.71, 0.57,
     $ 2.07, 1.89, 1.71, 1.46, 1.21, 1.08, 0.96, 0.75, 0.65, 0.53,
     $ 30*0.0/
      DATA Z2/
     $ 3.53, 3.86, 4.11, 4.64, 4.83, 4.94, 4.89, 4.40, 3.37, 3.11,
     $ 3.28, 3.45, 3.60, 3.73, 3.61, 3.55, 3.25, 2.95, 2.57, 2.48,
     $ 3.12, 3.24, 3.29, 3.30, 3.15, 2.93, 2.59, 2.29, 2.08, 1.81,
     $ 2.71, 2.65, 2.62, 2.41, 2.03, 1.75, 1.47, 1.28, 1.13, 0.99,
     $ 2.34, 2.18, 2.01, 1.70, 1.30, 1.07, 1.87, 0.76, 0.67, 0.56,
     $ 1.97, 1.72, 1.46, 1.13, 0.79, 0.62, 0.47, 0.38, 0.31, 0.26,
     $ 1.61, 1.22, 0.97, 0.69, 0.45, 0.36, 0.29, 0.22, 0.18, 0.15,
     $ 30*0.0/
     DATA Z3/
     $ 0.35, 0.49, 0.64, 0.95, 1.54, 2.05, 2.63, 3.35, 3.38, 3.11,
     $ 0.52, 0.69, 0.90, 1.24, 1.77, 2.09, 2.37, 2.55, 2.57, 2.48,
     $ 0.61, 0.82, 1.01, 1.35, 1.83, 2.08, 2.19, 2.12, 2.01, 1.79,
     $ 0.81, 1.00, 1.19, 1.43, 1.59, 1.57, 1.43, 1.27, 1.13, 0.99,
     $ 0.93, 1.09, 1.19, 1.26, 1.18, 1.04, 0.87, 0.76, 0.67, 0.56,
     $ 0.98, 1.07, 1.05, 0.97, 0.76, 0.62, 0.48, 0.39, 0.32, 0.27,
     $ 0.93, 0.89, 0.80, 0.64, 0.45, 0.36, 0.29, 0.22, 0.18, 0.15,
     $ 30*0.0/
      WRITE (6,100)
100
      FORMAT(* CENTER CRACK PANEL --- PLANE STRESS*,//)
      ITYPE=1
      NUMAB=7
      NUMN=10
      IFLAG=1
      RETURN
      END
```

```
SUBROUTINE CCPSTN
     COMMON/D/X(10),Y(10),Z1(100),Z2(100),Z3(100),IFLAG,METENG
     COMMON/F/NUMAB, NUMN, ITYPE
     DIMENSION XX(7), XY(7), XZ1(49), XZ2(49), XZ3(49)
     DATA XX/ 1.,1.5,2.,3.,5.,7.,10./
     DATA XY/ 0.25,0.3,0.4,0.5,0.6,0.7,0.75/
     DATA XZ1/
    $ 2.570, 2.900, 3.110, 3.350, 3.490, 3.430, 3.230,
    $ 2.450, 2.720, 2.890, 3.100, 3.110, 2.950, 2.720,
    $ 2.300, 2.450, 2.550, 2.600, 2.450, 2.230, 1.950,
    $ 2.190, 2.250, 2.270, 2.180, 1.930, 1.710, 1.440,
    $ 2.120, 2.060, 2.030, 1.830, 1.530, 1.350, 1.060,
    $ 2.100, 1.900, 1.870, 1.630, 1.230, 1.110, 0.840,
    $ 2.100, 1.850, 1.800, 1.570, 1.240, 1.040, 0.799/
     DATA XZ2/
     $ 2.790, 2.990, 3.090, 3.140, 3.000, 2.790, 2.470,
     $ 2.630, 2.800, 2.820, 2.790, 2.560, 2.280, 1.970,
     $ 2.350, 2.380, 2.320, 2.160, 1.800, 1.520, 1.270,
     $ 2.090, 1.990, 1.870, 1.610, 1.230, 0.996, 0.776,
     $ 1.810, 1.580, 1.420, 1.140, 1.780, 0.610, 0.450,
     $ 1.540, 1.230, 1.050, 0.630, 0.490, 0.360, 0.250,
     $ 1.400, 1.080, 0.899, 0.637, 0.401, 0.295, 0.204/
     DATA XZ3/
     $ 0.548, 0.752, 0.942, 1.270, 1.730, 1.970, 2.070,
     $ 0.600, 0.830, 1.010, 1.290, 1.620, 1.770, 1.750,
     $ 0.710, 0.920, 1.070, 1.260, 1.380, 1.350, 1.190,
     $ 0.798, 0.949, 1.060, 1.150, 1.100, 0.959, 0.771,
     $ 0.810, 0.900, 1.000, 0.970, 0.810, 0.600, 0.600,
     $ 0.820, 0.820, 0.780, 0.720, 0.530, 0.360, 0.360,
     $ 0.814, 0.763, 0.733, 0.593, 0.399, 0.294, 0.204/
      WRITE (6,100)
      FORMAT (* CENTER CRACK PANEL --- PLANE STRAIN*,//)
100
      DO 10 I=1,7
      X(I) = XX(I)
      Y(I) = XY(I)
      CONTINUE
10
      DO 20 I=1,49
      Z1(I) = XZ1(I)
      Z2(I) = XZ2(I)
      Z3(I) = XZ3(I)
20
      CONTINUE
      ITYPE=1
      IFLAG=1
      NUMAB=7
      NUMN=7
      RETURN
      END
```

```
SUBROUTINE CTSTS
     COMMON/D/X(10),Y(10),Z1(100),Z2(100),Z3(100),IFLAG,METENG
     COMMON/F/NUMAB, NUMN, ITYPE
      DIMENSION XX(10), XY(10), XZ1(72), XZ2(72), XZ3(72)
      DATA XX/1.,2.,3.,5.,7.,10.,13.,16.,20.,0.0/
      DATA XY/0.25,0.30,0.375,0.45,0.50,0.625,0.75,1.00,0.0,0.0/
      DATA XZ1/
     $ 1.61, 1.46, 1.28, 1.06, 0.90, 0.73, 0.60, 0.51, 0.39,
     $ 1.60, 1.35, 1.24, 0.94, 0.79, 0.59, 0.49, 0.39, 0.28,
      1.55, 1.25, 1.05, 0.80, 0.65, 0.48, 0.38, 0.28, 0.22,
       1.47, 1.11, 0.93, 0.78, 0.58, 0.46, 0.35, 0.24, 0.20,
      1.40, 1.08, 0.90, 0.69, 0.56, 0.44, 0.36, 0.30, 0.24,
     $ 1.27, 1.03, 0.88, 0.69, 0.59, 0.49, 0.42, 0.37, 0.31,
     $ 1.23, 0.98, 0.83, 0.68, 0.60, 0.51, 0.43, 0.37, 0.31,
     $ 1.13, 1.01, 0.78, 0.68, 0.65, 0.62, 0.49, 0.47, 0.42/
     DATA XZ2/
     $17.55,12.04,10.71, 8.74, 7.32, 5.74, 4.63, 3.75, 2.92,
     $15.10,10.27, 8.70, 6.80, 5.10, 3.90, 3.31, 2.20, 1.71,
     $12.41, 8.20, 6.54, 4.56, 3.45, 2.44, 1.83, 1.36, 1.02, $10.31, 6.50, 5.02, 3.32, 2.51, 1.73, 1.32, 1.09, 0.82,
     $ 9.16, 5.67, 4.21, 2.80, 2.12, 1.57, 1.25, 1.03, 0.81,
     $ 7.47, 4.48, 3.35, 2.37, 1.92, 1.54, 1.29, 1.12, 0.93,
     $ 6.25, 3.78, 2.89, 2.14, 1.78, 1.44, 1.20, 1.03, 0.86,
     $ 5.29, 3.54, 2.41, 1.91, 1.73, 1.59, 1.23, 1.17, 1.03/
      DATA XZ3/
     $ 9.67, 8.00, 7.21, 5.94, 5.00, 3.95, 3.19, 2.59, 2.02,
     $ 8.80, 7.10, 6.10, 6.88, 3.60, 2.82, 2.22, 1.69, 1.30,
     $ 7.80, 5.73, 4.62, 3.25, 2.48, 1.77, 1.33, 0.99, 0.75,
     $ 6.68, 4.73, 3.68, 2.42, 1.81, 1.27, 1.00, 1.80, 0.60,
     $ 6.29, 4.15, 3.11, 2.09, 1.59, 1.18, 0.94, 0.77, 0.61,
     $ 5.42, 3.38, 2.54, 1.80, 1.47, 1.18, 0.99, 0.85, 0.71,
     $ 4.77, 2.92, 2.24, 1.66, 1.38, 1.12, 0.94, 0.80, 0.67,
     $ 4.23, 2.83, 1.93, 1.52, 1.39, 1.27, 0.99, 0.93, 0.82/
      WRITE (6,100)
      FORMAT(* COMPACT TENSION SPECIMEN --- PLANE STRESS*,//)
100
      ITYPE=2
      NUMAB=8
      NUMN=9
      DO 1 I=1,10
      X(I) = XX(I)
      Y(I) = XY(I)
1
      DO 2 I=1.72
      Z1(I) = XZ1(I)
      Z2(I) = XZ2(I)
      Z3(I) = XZ3(I)
2
      IFLAG=1
      RETURN
      END
```

```
SUBROUTINE CTSTN
     COMMON/D/X(10),Y(10),Z1(100),Z2(100),Z3(100),IFLAG,METENG
     COMMON/F/NUMAB, NUMN, ITYPE
     DIMENSION XX(10), XY(10), XZ1(63), XZ2(63), XZ3(63)
     DATA XX/1.,2.,3.,5.,7.,10.,13.,16.,20.,0.0/
     DATA XY/0.25,0.3,0.375,0.45,0.5,0.625,0.75,3*0.0/
     DATA XZ1/
    $ 2.227, 2.048, 1.783, 1.475, 1.334, 1.248, 1.258, 1.325, 1.566,
    $ 2.200, 1.870, 1.540, 1.190, 0.960, 0.780, 0.700, 0.500, 0.520,
    $ 2.148, 1.716, 1.392, 0.970, 0.693, 0.443, 0.276, 0.176, 0.098,
    $ 2.040, 1.580, 1.280, 0.900, 0.540, 0.390, 0.280, 0.200, 0.060,
    $ 1.935, 1.509, 1.242, 0.919, 0.685, 0.461, 0.314, 0.216, 0.132,
    $ 1.763, 1.449, 1.237, 0.974, 0.752, 0.602, 0.459, 0.347, 0.248,
    $ 1.709, 1.424, 1.263, 1.033, 0.864, 0.717, 0.575, 0.448, 0.345/
     DATA XZ2/
    $17.883,12.481,11.675,10.788,10.538,10.745,11.460,12.570,14.563,
    $15.850,10.430, 8.900, 6.020, 5.250, 3.840, 3.700, 7.750, 9.200,
    $12.644, 8.176, 6.521, 4.319, 2.970, 1.744, 1.102, 0.686, 0.370,
    $10.550, 6.650, 5.030, 3.180, 2.200, 0.820, 0.830, 0.520, 0.310,
    $ 9.327, 5.846, 4.304, 2.747, 1.912, 1.199, 0.788, 0.530, 0.317,
    $ 7.612, 4.572, 3.423, 2.359, 1.810, 1.319, 0.983, 0.749, 0.485,
    $ 6.370, 3.948, 3.179, 2.337, 1.876, 1.441, 1.124, 0.887, 0.685/
     DATA XZ3/
    $ 9.852, 8.506, 8.170, 7.774, 7.706, 7.942, 8.517, 9.371,10.887,
    $ 9.000, 7.100, 4.170, 4.660, 3.540, 3.050, 2.600, 5.700, 6.700,
    $ 7.944, 5.760, 4.643, 3.103, 2.139, 1.292, 0.793, 0.494, 0.266,
    $ 7.020, 4.730, 3.560, 2.360, 1.630, 0.910, 0.580, 0.360, 0.200,
    $ 6.406, 4.268, 3.157, 2.024, 1.413, 0.888, 0.585, 0.393, 0.236,
     $ 5.521, 3.431, 3.179, 1.787, 1.373, 1.000, 0.746, 0.568, 0.368,
     $ 4.857, 3.048, 2.456, 1.807, 1.450, 1.114, 0.869, 0.686, 0.514/
      IFLAG=1
      ITYPE=2
      DO 10 I=1,10
      X(I) = XX(I)
      Y(I) = XY(I)
10
      DO 20 I=1,63
      Z1(I) = XZ1(I)
      Z2(I) = XZ2(I)
20
      Z3(I) = XZ3(I)
      NUMAB=7
      NUMN=9
      WRITE (6,100)
      FORMAT(* COMPACT TENSION --- PLANE STRAIN*,//)
100
      RETURN
      END
```

```
SUBROUTINE DCPSTN
     COMMON/D/X(10),Y(10),Z1(100),Z2(100),Z3(100),IFLAG,METENG
     COMMON/F/NUMAB, NUMN, ITYPE
     DIMENSION XX(9), XY(9), XZ1(45), XZ2(45), XZ3(45)
     DATA XX/1., 2., 3., 5., 7., 10., 13., 16., 20./
     DATA XY/0.25, 0.375, 0.5, 0.625, 0.75, 4*0.0/
      DATA XZ1/
     $ 5.01,128.4,28.67,155.43,749.07,7530.8,75618.,741530.,156.E5,
     $ 6.41,14.56,30.42,120.99,461.24,3354.2,20374.,168810.,2278.E3,
     $ 7.31,14.59,27.07,87.53,275.57,1505.9,8109.4,43102.,343780.,
     $ 7.87,13.40,21.62,53.51,128.26,455.26,1558.4,5376.8,28037.,
     $ 8.19,12.4,17.72,33.06,55.12,105.32,221.79,518.,894.59/
     DATA XZ2/
     $ 3.33,7.41,16.55,78.46,362.04,3507.8,34044.,326360.,6719300.,
     $ 5.00,9.61,18.64,67.83,244.2,1676.3,11491.,80300.,1062000.,
     $ 6.78,11.22,18.94,53.76,155.16,784.89,4008.9,20719.,173640.,
     $ 8.79,11.91,16.73,34.22,73.4,238.5,783.6,2623.4,13141.,
     $ 11.37,12.21,14.07,20.72,31.92,58.76,117.99,262.33,473.82/
     DATA XZ3/
     $ 0.57,2.2,6.82,49.33,297.75,3720.9,43022.,459780.,10421.E3,
     $ 1.27,4.22,11.17,59.32,266.63,2211.5,16698.,123400.,1693100.,
     $ 2.28,6.39,14.29,55.49,187.17,1051.1,5610.1,29420.,244640.,
     $ 3.73,8.28,14.83,38.63,91.15,310.61,1030.7,3458.7,17316.,
     $ 5.91,9.89,13.89,23.82,38.01,70.98,142.31,315.28,575.54/
      DO 10 I=1,9
      X(I) = XX(I)
      Y(I) = XY(I)
10
      CONTINUE
      DO 20 I=1,45
      Z1(I) = XZ1(I)
      Z2(I) = XZ2(I)
      Z3(I) = XZ3(I)
20
      CONTINUE
      WRITE(6,100)
100
      FORMAT(* DOUBLE-EDGE CRACK PANEL --- PLANE STRAIN*,//)
      ITYPE=3
      IFLAG=1
      NUMAB=5
      NUMN=9
      RETURN
      END
```

```
SUBROUTINE STBSTN
     COMMON/D/X(10),Y(10),Z1(100),Z2(100),Z3(100),IFLAG,METENG
     COMMON/F/NUMAB, NUMN, ITYPE
     DIMENSION XX(10), XY(10), XZ1(63), XZ2(63), XZ3(63)
     DATA XX/1.,2.,3.,5.,7.,10.,13.,16.,20.,0.0/
     DATA XY/0.25,0.3,0.35,0.4,0.5,0.6,0.75,3*0.0/
     DATA XZ1/
    $ 1.195, 1.034, 0.930, 0.765, 0.633, 0.523, 0.396, 0.303, 0.215,
    $ 1.270, 1.040, 0.920, 0.740, 0.570, 0.450, 0.320, 0.240, 0.150,
    $ 1.340, 1.050, 0.910, 0.720, 0.540, 0.410, 0.280, 0.200, 0.120,
    $ 1.370, 1.070, 0.910, 0.700, 0.510, 0.370, 0.240, 0.160, 0.090,
    $ 1.398, 1.094, 0.922, 0.675, 0.495, 0.331, 0.211, 0.135, 0.074,
    $ 1.360, 1.100, 0.920, 0.670, 0.480, 0.320, 0.250, 0.110, 0.150,
    $ 1.208, 1.145, 0.974, 0.693, 0.500, 0.348, 0.223, 0.140, 0.075/
     DATA XZ2/
    $ 5.799, 4.665, 4.006, 3.080, 2.454, 1.934, 1.446, 1.088, 0.758,
    $ 5.520, 4.180, 3.450, 2.670, 2.060, 1.580, 1.100, 0.830, 0.550,
    $ 5.300, 4.620, 3.120, 2.330, 1.760, 1.300, 0.870, 0.650, 0.410,
    $ 5.140, 3.630, 2.860, 2.050, 1.520, 1.080, 0.700, 0.520, 0.300,
    $ 4.869, 3.283, 2.527, 1.686, 1.192, 0.773, 0.480, 0.304, 0.165,
    $ 4.650, 3.030, 2.300, 1.440, 0.980, 0.630, 0.380, 0.250, 0.110,
    $ 4.474, 2.754, 2.096, 1.361, 0.936, 0.618, 0.388, 0.239, 0.128/
     DATA XZ3/
    $ 4.083,10.099, 8.413, 5.864, 4.466, 3.421, 2.542, 1.901, 1.318,
    $ 4.280, 7.600, 6.350, 4.290, 3.340, 2.470, 1.590, 1.250, 0.900,
    $ 4.440, 6.330, 5.140, 3.540, 2.740, 1.920, 1.230, 0.930, 0.650,
    $ 4.560, 5.500, 4.420, 3.050, 2.290, 1.550, 0.980, 0.700, 0.470,
    $ 4.687, 4.442, 3.509, 2.353, 1.663, 1.079, 0.669, 0.424, 0.230,
    $ 4.670, 3.800, 2.300, 1.900, 1.320, 1.830, 0.510, 0.280, 0.140,
    $ 1.491, 3.159, 2.407, 1.557, 1.068, 0.704, 0.441, 0.272, 0.144/
     DO 10 I=1,10
     X(I) = XX(I)
     Y(I) = XY(I)
10
     CONTINUE
      DO 20 I=1,63
      Z1(I) = XZ1(I)
      Z2(I) = XZ2(I)
      Z3(I) = XZ3(I)
20
      CONTINUE
      WRITE (6,100)
      FORMAT(* SINGLE EDGE CRACK PANEL IN 3-POINT BENDING ---*
100
     $ * PLANE STRAIN*,//)
      ITYPE=4
      IFLAG=1
      NUMAB=7
      NUMN=9
      RETURN
      END
```

```
SUBROUTINE SETSTN
      COMMON/A/E, EA, PLCOR, SIGO, XLL, B, FACTOR, XN, ALPHA, EPO, TH, PP1, IPL,
     $A,H1,H2,H3,PP,ZZ,ZZE,PO,AE,XKSQ,XJ,BETA,PI
      COMMON/D/X(10),Y(10),Z1(100),Z2(100),Z3(100),IFLAG,METENG
      COMMON/E/PMIN, PMAX, PIN, ZZO, AIN, DISMAX
      COMMON/F/NUMAB, NUMN, ITYPE
      DIMENSION XX(7), XY(7), XZ1(42), XZ2(42), XZ3(42)
      DATA XX/1.,2.,3.,5.,7.,10.,0.0/
      DATA XY/0.25,0.3,0.35,0.4,0.5,0.6,0.75/
      DATA XZ1/
     $ 4.338, 4.768, 4.639, 3.815, 3.056, 2.170,
     $ 4.200, 3.880, 3.250, 2.520, 2.033, 1.490,
     $ 4.020, 3.340, 2.650, 1.900, 1.450, 1.010,
     $ 3.820, 3.030, 2.230, 1.460, 1.040, 0.660,
     $ 3.398, 2.302, 1.694, 0.928, 0.514, 0.213,
     $ 2.960, 1.910, 1.400, 1.690, 0.330, 0.110,
     $ 2.342, 1.607, 1.245, 0.769, 0.477, 0.233/
      DATA XZ2/
     $ 4.756, 4.559, 4.281, 3.391, 2.639, 1.808,
     $ 4.660, 3.670, 3.350, 2.310, 1.790, 1.260,
     $ 4.590, 3.280, 2.790, 1.750, 1.270, 0.870,
     $ 4.530, 3.030, 2.230, 1.870, 0.910, 0.580,
     $ 4.447, 2.765, 1.888, 0.954, 0.507, 0.204,
     $ 4.880, 2.520, 1.400, 0.810, 0.380, 0.150,
     $ 4.316, 2.515, 1.789, 1.027, 0.619, 0.296/
      DATA XZ3/
     $10.270, 7.635, 5.874, 3.695, 2.483, 1.496,
     $ 8.150, 4.120, 3.000, 1.930, 1.560, 0.900,
     $ 6.050, 3.000, 1.880, 1.290, 1.030, 0.580,
     $ 4.600, 2.370, 1.350, 0.880, 0.590, 0.350,
     $ 3.151, 1.537, .9125, .4172, .2151, .0851,
     $ 2.520, 1.150, 0.750, 0.500, 0.060, 0.040,
     $ 2.018, 1.105, .7655, .4349, .2617, .1254/
      WRITE(6,100)
100
      FORMAT(* SINGLE EDGE CRACK PANEL IN TENSION --- *
     $ *PLANE STRAIN*,//)
      ITYPE=5
      IFLAG=1
      NUMAB=7
      NUMN=6
      DO 10 I=1,7
      X(I) = XX(I)
      Y(I) = XY(I)
10
      CONTINUE
      DO 20 I=1,42
      Z1(I) = XZ1(I)
      Z2(I) = XZ2(I)
      Z3(I) = XZ3(I)
20
      CONTINUE
      RETURN
      END
```

```
SUBROUTINE DCPSTS
     COMMON/D/X(10),Y(10),Z1(100),Z2(100),Z3(100),IFLAG,METENG
     COMMON/F/NUMAB, NUMN, ITYPE
     DIMENSION XX(9), XY(9), XZ1(45), XZ2(45), XZ3(45)
     DATA XX/1.,2.,3.,5.,7.,10.,13.,16.,20./
     DATA XY/0.25,0.37,0.5,0.62,0.75,0.0,0.0,0.0,0.0/
     DATA XZ1/
    $ 1.011, 1.226, 1.356, 1.483, 1.543, 1.578, 1.594, 1.591, 1.588,
    $ 1.293, 1.418, 1.427, 1.341, 1.237, 1.094, 0.970, 0.873, 0.674,
    $ 1.475, 1.466, 1.378, 1.168, 1.010, 0.845, 0.732, 0.625, 0.208,
     $ 1.586, 1.454, 1.284, 1.038, 0.882, 0.737, 0.649, 0.466, 0.020,
    $ 1.652, 1.425, 1.118, 0.979, 0.833, 0.701, 0.630, 0.297, 0.000/
     DATA XZ2/
    $ 1.726, 1.819, 1.886, 1.917, 1.905, 1.853, 1.802, 1.746, 1.700,
     $ 2.594, 2.393, 2.221, 1.864, 1.588, 1.283, 1.068, 0.922, 0.709,
     $ 3.514, 2.821, 2.337, 1.670, 1.277, 0.944, 0.762, 0.630, 0.232,
     $ 4.559, 3.145, 2.318, 1.449, 1.061, 0.790, 0.657, 0.473, 0.028,
     $ 5.896, 3.371, 2.214, 1.297, 0.966, 0.741, 0.634, 0.312, 0.000/
     DATA XZ3/
     $ 0.295, 0.537, 0.770, 1.169, 1.490, 1.815, 2.022, 2.124, 2.198,
     $ 0.658, 1.037, 1.295, 1.520, 1.547, 1.412, 1.227, 1.068, 0.829,
     $ 1.184, 1.581, 1.691, 1.563, 1.320, 0.080, 0.808, 0.662, 0.265,
     $ 1.932, 2.138, 1.950, 1.444, 1.094, 0.809, 0.664, 0.487, 0.032,
     $ 3.063, 2.670, 2.061, 1.314, 0.978, 0.747, 0.638, 0.318, 0.000/
      WRITE (6,100)
      FORMAT(* DOUBLE EDGE CRACK PANEL --- PLAIN STRESS*,//)
100
      ITYPE=3
      IFLAG=1
      NUMAB=5
      NUMN=9
      DO 10 I=1,9
      X(I) = XX(I)
      Y(I) = XY(I)
10
      CONTINUE
      DO 20 I=1,45
      Zl(I) = XZl(I)
      Z2(I) = XZ2(I)
      Z3(I) = XZ3(I)
20
      CONTINUE
      RETURN
      END
```

```
SUBROUTINE STBSTS
      COMMON/D/X(10),Y(10),Z1(100),Z2(100),Z3(100),IFLAG,METENG
      COMMON/F/NUMAB, NUMN, ITYPE
      DIMENSION XX(9), XY(9), XZ1(45), XZ2(45), XZ3(45)
      DATA XX/1.,2.,3.,5.,7.,10.,13.,16.,20./
      DATA XY/0.25,0.37,0.5,0.62,0.75,0.0,0.0,0.0,0.0/
      DATA XZ1/
     $ 0.869, 0.731, 0.629, 0.479, 0.370, 0.246, 0.174, 0.117, 0.059,
     $ 0.963, 0.797, 0.680, 0.527, 0.418, 0.307, 0.232, 0.174, 0.105,
     $ 1.019, 0.767, 0.621, 0.453, 0.324, 0.202, 0.128, 0.081, 0.030,
     $ 1.051, 0.786, 0.649, 0.494, 0.357, 0.235, 0.173, 0.105, 0.047,
     $ 1.067, 0.786, 0.643, 0.474, 0.343, 0.230, 0.167, 0.110, 0.044/
      DATA XZ2/
     $ 5.690, 4.503, 3.680, 2.614, 1.947, 1.290, 0.897, 0.603, 0.307,
     $ 5.085, 3.732, 2.929, 2.071, 1.580, 1.134, 0.841, 0.626, 0.381, $ 4.768, 3.120, 2.320, 1.547, 1.077, 0.655, 0.410, 0.259, 0.097,
     $ 4.551, 2.830, 2.118, 1.455, 1.023, 0.656, 0.472, 0.286, 0.130,
     $ 4.385, 2.656, 1.967, 1.329, 0.928, 0.601, 0.427, 0.280, 0.114/
      DATA XZ3/
     $ 4.007, 8.812, 7.189, 4.731, 3.388, 2.204, 1.517, 1.012, 0.508,
     $ 4.420, 5.533, 4.482, 3.172, 2.409, 1.726, 1.277, 0.948, 0.575,
     $ 4.604, 4.085, 3.092, 2.081, 1.442, 0.874, 0.545, 0.344, 0.129,
     $ 4.617, 3.434, 2.599, 1.794, 1.258, 0.803, 0.577, 0.349, 0.158,
     $ 4.394, 3.012, 2.235, 1.510, 1.052, 0.680, 0.483, 0.316, 0.129/
      DO 10 I=1,9
      X(I) = XX(I)
      Y(I) = XY(I)
10
      CONTINUE
      DO 20 I=1,45
      Z1(I) = XZ1(I)
      Z2(I) = XZ2(I)
      Z3(I) = XZ3(I)
20
      CONTINUE
      WRITE (6,100)
      FORMAT(* SINGE EDGE CRACK PANEL IN 3-POINT BENDING ---*
100
     $ *PLANE STRESS*,//)
      ITYPE=4
      IFLAG=1
      NUMAB=5
      NUMN=9
      RETURN
      END
```

```
SUBROUTINE SETSTS
      COMMON/D/X(10),Y(10),Z1(100),Z2(100),Z3(100),IFLAG,METENG
      COMMON/F/NUMAB, NUMN, ITYPE
      DIMENSION XX(9), XY(9), XZ1(45), XZ2(45), XZ3(45)
      DATA XX/1..2..3..5..7..10..13..16..20./
      DATA XY/0.25,0.37,0.5,0.62,0.75,0.0,0.0,0.0,0.0/
      DATA XZ1/
     $ 3.140, 3.261, 2.919, 2.115, 1.531, 0.960, 0.615, 0.400, 0.230,
     $ 2.809, 2.365, 1.943, 1.367, 1.009, 0.677, 0.474, 0.342, 0.226,
     $ 2.459, 1.665, 1.254, 0.776, 0.510, 0.286, 0.164, 0.096, 0.047,
     $ 2.070, 1.408, 1.105, 0.755, 0.551, 0.363, 0.248, 0.172, 0.107,
     $ 1.696, 1.142, 0.910, 0.624, 0.447, 0.280, 0.181, 0.118, 0.067/
      DATA XZ2/
     $ 4.672, 4.300, 3.695, 2.532, 1.755, 1.053, 0.656, 0.419, 0.237,
     $ 4.465, 3.426, 2.632, 1.685, 1.181, 0.762, 0.524, 0.372, 0.244,
     $ 4.369, 2.726, 1.909, 1.093, 0.694, 0.380, 0.216, 0.124, 0.061,
     $ 4.297, 2.552, 1.837, 1.160, 0.816, 0.523, 0.353, 0.242, 0.150,
     $ 4.240, 2.468, 1.805, 1.147, 0.798, 0.490, 0.314, 0.203, 0.115/
      DATA XZ3/
     $10.090, 6.488, 4.362, 2.185, 1.239, 0.630, 0.362, 0.224, 0.123,
     $ 5.047, 2.653, 1.604, 0.812, 0.525, 0.328, 0.223, 0.157, 0.102,
     $ 3.095, 1.429, 0.871, 0.461, 0.286, 0.155, 0.088, 0.051, 0.025,
     $ 2.270, 1.127, 0.771, 0.478, 0.336, 0.215, 0.146, 0.100, 0.062,
     $ 1.983, 1.087, 0.784, 0.494, 0.344, 0.211, 0.136, 0.058, 0.050/
      WRITE (6,100)
      FORMAT(* SINGLE EDGE CRACK PANEL IN TENSION ---*
100
     $ *PLANE STRESS*,//)
      ITYPE=5
      IFLAG=1
      NUMAB=5
      NUMN=9
      DO 10 I=1,9
      X(I) = XX(I)
      X(I) = XY(I)
10
      CONTINUE
      DO 20 I=1,45
      Z1(I) = XZ1(I)
      Z2(I) = XZ2(I)
      Z3(I) = XZ3(I)
20
      CONTINUE
      RETURN
      END
```

APPENDIX B

A SAMPLE OUTPUT LISTING OF PROGRAM EST

(INPUT IS DEFINED IN APPENDIX C)

..COPY,TAPE6 ..COPY,TAPE6 MATERIAL: COPPER COMPACT TENSION --- PLANE STRAIN

EPO	.1941E-02			0.	.124E	.123E-05 .130E-81	.136E	.118E	.991E	.143E	.151E	925E	.476E	.236E	.104E-6	.413E-6	S-03 . 150E	111E-04 .158E-6	7E-04 .46/E-0				EC PC	.0 0.	.794E-06 .241E-86	.159E-05 .253E-80	.238E-05 .840E-11	307F-05 230F-72	. 477E-05 .881E-71	.556E-05 .192E-69	.635E-05 .278E-68	.715E-05 .293E-67	500	0.5	04	.111E-04 .201E-63	40	
ALPHA	1.030			N.																			PN										•					
				EN	0		0	0	00	00	0	0	0	00	0	0	00	0	0				EN	0	0	0	0	0 0	5 6		0	0	0				_	
IC	1.000			LDISP	617R-06 0.	-05 0	247E-05 0.	0	-05 0	050	0 20	05 0	020	803E-05 0.	-050	-05 0	.988E-05 0.	E-04 0	.117E-04 0.				LDISP	0.	.794E-06 0.	59E-05	0	0	0	0	0	0	0	.874E-05 0.		1E-04 0	.119E-04 0.	
WIDTH	1.995			CMOD		.229E-05											.183E-04	9	.217E-04				CMOD		137R-05	.274E-05									179E-04	0	.206E-04	
M	.1700E+08			SQRT (J/JE)	0000000	1.00000000	1.0000000	1.0000000	1.0000000	1.00000000	1.0000001	1.0000001	1.0000001	1.0000001	.000000	.0000002	1.0000002	1.0000002	.000000				(91/1/10)		000000	1.0000000	1.0000000	1	-		7 -			-		1.000000	÷.	
PZC	1.000	10		SQRT (EJ)	0.000	.700E+01	.105E+02	.140E+02	.210E+02	.245E+02	315E+02	.350E+02	.385E+02	.420E+02	455E+02	525E+02	.560E+02	.595E+02	.665E+02		392		TEN MOOD	SURI (EU)	V. AODETOI	799E+01	.120E+02				320E+02					.560E+02	.600E+02	
SIGO	.3300E+05 1	10.8870		ŋ		.656E-06	[2]	田田		田口	.420E-04	E	.794E-04	.945E-04	.111E-03	148E-03	.168E-03	.190E-03	.237E-03		6.6892			ר י	C	342E-05	770E-05	.137E-04	.214E-04	.308E-04	.419E-04	693E-04	855E-04	.103E-03	.123E-03	145E-03	.192E-03	
IS	.3300	500 0 H3 =		×		.350E+01	105E+0	.140E+02	210E+0	245E+0	280E+0	350E+02	.385E+02	.420E+02	5E+0	490E+C	+	5E+(5E+(3001	52 H3 =			×		.400E+01	NO C	60E	OOE	40E	80E	350E+02	OOF	40E	480E	.520E+02	OF	
z	36.00	= .2500 14.5630		AD	0	1.000	1.500	2.000	3.000	3.500	4.000	000	.500	000.	6.500	7.000	8.000	8.500	9.500	"	9.18																7.500	
N.	4.000	A/B H2 =	IN ENGLISH UNITS	P/P0		.505E-04	0	.202E-03		0	0	D C	, ,	0	656E-0	06E-0	07E-(858E-(.908E-03	A/B	H2 =	IN ENGLISH UNITS		P/P0		594E-0	119E-0	1/8E-0	0	356E-0	416E-0	475E-0	9 0	594E-0	713E-0	0 0	.832E-03	
SPAN	4.	1.5660	IN ENG	AE	0	499	0	499	499	499	66	499	ח ס	499	4	011	.499	01	.499	20.00	.5193			AE	6	59	59	50	59	59	59	000	59	59	59	59	. 599	
	80	20	RESULTS	•	10	.250	7 10	25	\cap	25	25	25	2 6	1 4 1	2	5	.250	25	.250	2	1	RESULTS		N/W	.3	•		•	•	• •	•	•	•	300		.3	.300	
H	.4980	H Z	ALL RE	A	.499	4	499	4.	499	L 4	4	4.	4 4	.499	4	.499	499	4	.499	11	-	ALL RE		A	0	0	59	59	200	0	59	6	59	90	59	59	. 599	

					_
	.291E-62 .979E-62 .307E-61			PC 343E-78 343E-78 3114E-75 3120E-69 320E-71 320E-69 327E-65 327E-65 327E-65 327E-65 327E-67 326E-61 3398E-66 327E-67 3398E-66 337E-67 3398E-66 337E-67 3398E-66 3398E-66 3398E-66 3398E-66 3398E-67 3398E	2-5
	.127E-04 .135E-04 .143E-04			EC 00 1018 - 05	3E-0
				Na N	
	.127E-04 0. .135E-04 0. .143E-04 0.			LDISP 0.101E-05 0.202E-05 0.303E-05 0.303E-05 0.303E-05 0.303E-05 0.303E-05 0.303E-05 0.303E-04 0.303E-04 0.303E-05 0.303E-05 0.303E-05 0.303E-05 0.303E-05 0.303E-05 0.303E-05 0.303E-05 0.303E-05 0.303E-04 0.303B-04	-04
	.220E-04 .233E-04 .247E-04			CMOD 0. 164E-05 . 328E-05 . 652E-05 . 652E-05 . 652E-05 . 652E-05 . 131E-04 . 131E-04 . 1393E-04 . 246E-04 . 246E-04 . 246E-04 . 246E-05 . 983E-05 . 983E-05 . 983E-05 . 138E-04 . 216E-04	04
	1.0000002 1.0000003 1.0000003			SQRT(J/JE) 0.00000000 1.00000000 1.00000000 1.00000000	1.000000
	.639E+02 .679E+02 .719E+02	892		SQRT(EJ) . 455E+01 . 909E+01 . 136E+02 . 273E+02 . 273E+02 . 318E+02 . 364E+02 . 590E+01 . 136E+02 . 455E+02 . 590E+02 . 773E+02 . 773E+02 . 104E+02 . 155E+02 . 207E+02 . 311E+02 . 437 . 518E+01 . 155E+02 . 590E+02 . 590E+02 . 590E+02 . 570E+02	.984E+02
	.219E-03 .247E-03 .277E-03	2.38		J 1111E-05 443E-05 996E-05 1277E-04 1277E-04 1277E-04 1398E-04 1398E-03 1194E-03 1298E-04 1359E-03 1359E-04 1359E	18E-0
	.639E+02 .679E+02 .719E+02 .759E+02	.3503 838 H3 =		K 455E+01 909E+01 136E+02 227E+02 273E+02 318E+02 364E+02 364E+02 500E+02 500E+02 500E+02 500E+02 516E+02 516E+02 773E+02 637E+02 637E+02 637E+02 637E+02 773E+02 777E+02	84E+0
	8.000 8.500 9.000 9.500	3.21		LOAD 11.5000 12.5000 13.5000 14.5000 15.5000 16.5000 17.5000 18.500	
	.951E-03 .101E-02 .107E-02	A/B 3 H2 =	ENGLISH UNITS	0.7076- 1416- 1416- 14954- 49546- 49546- 17076- 17076- 19196- 11346- 11026- 11706- 117	162E-0
	.599	20.00	IN	I N I N I N I N I N I N I N I N I N I N	9
•	.300	2	RESULTS	2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	.400
	.599 .599 .599	N = H1 =	ALL RE	ALL RE.	20 00

APPENDIX C

A SAMPLE INPUT LISTING OF THE INTERACTIVE COMPUTER PROGRAM EST FOR A MONOTONICALLY LOADED CT SPECIMEN COMMAND-ATTACH, EST, ID=TUSIT, SN=AFML, CY=1 COMMAND-EST.

ENTER SPECIMEN GEOMETRY

(CCP,CT,DCP,STB,SET) == |CT

PLANE STRESS (0) OR PLANE STRAIN (1) ?1

MATERIAL == |COPPER

E(NGLISH) OR M(ETRIC) ?E

IF THE FOLLOWING VALUES ARE CORRECT, RESPOND WITH A "Y". IF THEY ARE INCORRECT, ANSWER "N", THEN ENTER THE CORRECT VALUE

MINIMUM LOAD (KIPS/N) = .0(Y/N)?Y LOAD INCREMENT (KIPS/N) = .25000(Y/N)?N (Y/N)?N MAXIMUM LOAD(KIPS/N) = 5.3INITIAL A/B = .25000(Y/N)?Y RAMBERG-OSGOOD EXPONENT - N = 36.000(Y/N)?Y YIELD STRENGTH - SIGO (KSI/PA) = 33000. (Y/N)?Y PLASTIC ZONE CORRECTION FACTOR - PZC= 1.0000 (Y/N)?Y ELASTIC MODULUS - E (KSI/PA) = .17000E+08 (Y/N)?Y (Y/N)?Y HALF SPAN (IN/M) = 2.0000WIDTH - W (IN/M) = 1.9950(Y/N)?Y THICKNESS - TH (IN/M) = .49800 (Y/N)?Y CRACK LENGTH INCREMENT (IN/M) = .10000 (Y/N)? CORR. FACTOR FOR LIMIT LOAD - LC = 1.0000 MAXIMUM DISPLACEMENT (IN/M) = .10000E-01 (Y/N)?N0.1

STOP 030400 MAXIMUM EXECUTION FL. .438 CP SECONDS EXECUTION TIME.